Mathematics with Computer Science

Introductory Course
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Technische Universität Darmstadt
Fachbereich Mathematik
Dennis Frisch

Hints to exercises 2

1. (a)

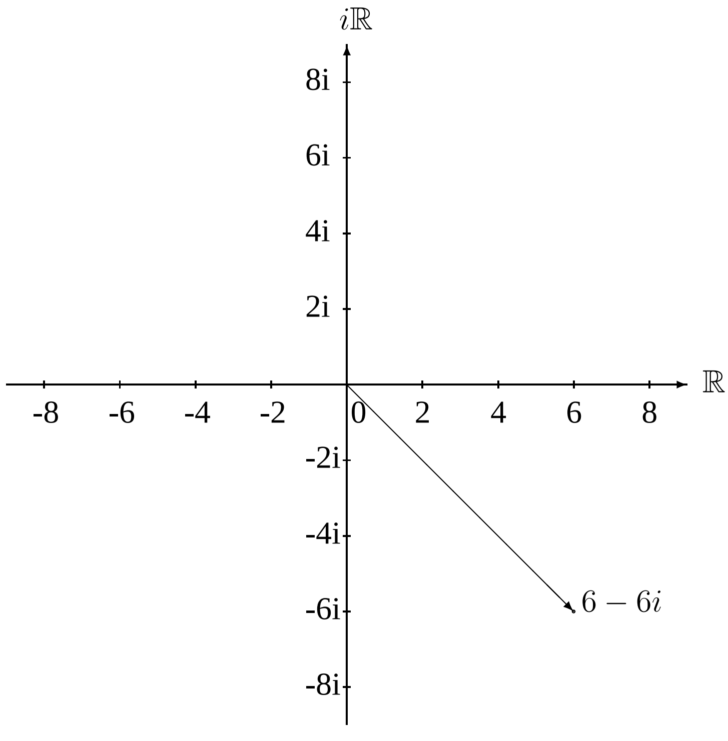
$$(a+bi)(a+bi)^{-1} = (a+bi)\cdot\left(\frac{a}{a^2+b^2} + \frac{-b}{a^2+b^2}i\right)$$
$$= a\frac{a}{a^2+b^2} + a\frac{-b}{a^2+b^2}i + bi\frac{a}{a^2+b^2} + b\frac{-b}{a^2+b^2}i^2$$
$$= \frac{a^2}{a^2+b^2} + \frac{b^2}{a^2+b^2} = 1$$

is correct.

(b)

$$\frac{5-3i}{3+2i} = (5-3i)(3+2i)^{-1} = (5-3i)\frac{3-2i}{13} = \frac{9-19i}{13}.$$

(c) We see, that $2-2i$ enclose an angel of $45°$ with the real line. So multiplicating $2-2i$ is a rotation of $45°$ clockwise and a dilation of $|2-2i| = \sqrt{8} \approx 2.8$. So we get



(d)

$$(x-c)(x-\bar{c}) = x^2 - (c+\bar{c})x + c\bar{c} = x^2 - 2ax + (a^2+b^2).$$

If $a = b = 1$ this turns into $x^2 - 2x + 2$ which hence has the solutions $1+i$ and $1-i$.

2. The truth tables are:

   (a)

A	$\neg A$	$\neg(\neg A)$
t	f	t
f	t	f

   (b)

A	B	$A \wedge B$	$\neg(A \wedge B)$	$\neg A$	$\neg B$	$\neg A \vee \neg B$
t	t	t	f	f	f	f
t	f	f	t	f	t	t
f	t	f	t	t	f	t
f	f	f	t	t	t	t

   (c) Similar to the above.

   (d)

A	B	$A \Rightarrow B$	B	$\neg A$	$\neg A \vee B$
t	t	t	t	f	t
t	f	f	f	f	f
f	t	t	t	t	t
f	f	t	f	t	t

3. By setting up a truth table.

   The implication $A \Rightarrow B$ says that B is true provided A is also true. It does not say anything about the truth of A and therefore, we do not know if B is true or not.

   If we know in addition that A is true, then it follows that B is also true.

4. If x is a positive real number, then x^2 is also positive.
   If the integer n is divisible by 6, then n is divisible by 3.
   If it is midnight in Darmstadt, the sun does not shine.

5. We are looking at the following truth table:

A	B	$A \Rightarrow B$	$\neg B$	$\neg A$	$\neg B \implies \neg A$
t	t	t	f	f	t
t	f	f	t	f	f
f	t	t	f	t	t
f	f	t	t	t	t

6. (a) There is a mathematician who is not a smoker.
   (b) There is a student who likes to go to parties.
   (c) There is a banana that is not yellow.
   (d) All swans are not black.

7. (a) $\forall x \in S : \neg A(x) \vee \neg B(x)$

(b) $\exists x \in S : A(x) \wedge \neg B(x)$

8. (a) $\forall x \in \mathbb{R}\ \exists n \in \mathbb{N} : n > x$.
   There is an x in $\mathbb{R}$ such that x is larger than or equal to n for all $n \in \mathbb{N}$.

   (b) $\neg(\exists q \in \mathbb{Q} : q^2 = 0)$ which is the same as $\forall q \in \mathbb{Q} : q^2 \neq 0$.
   There is a rational number x satisfying the equation $x^2 = 0$.