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$$\bullet \langle x, (qS)^* y \rangle = \langle rSx, y \rangle = q \langle Sx, y \rangle = q \langle x, S^* y \rangle = \langle x, \bar{q} S^* y \rangle$$

$$\text{für alle } x, y \in V \Rightarrow (rS)^* = \bar{q} S^*$$

$$\bullet \langle STx, y \rangle = \langle Tx, S^* y \rangle = \langle x, T^* S^* y \rangle$$

$$= \langle x, (ST)^* y \rangle \quad \forall x, y \in V$$

$$\bullet \text{ Sei } x \in \ker T \Leftrightarrow \forall y: \langle Tx, y \rangle = 0$$

$$\Leftrightarrow \forall y: \langle x, T^* y \rangle = 0$$

$$\Leftrightarrow x \perp \text{Bild } T^*$$

$$\bullet \lambda \in \sigma(T) \Leftrightarrow \det(T - \lambda \mathbb{1}) = 0 \Leftrightarrow \overline{\det(T - \lambda \mathbb{1})}$$

$$\Leftrightarrow \det((T - \lambda \mathbb{1})^*) = 0 \Leftrightarrow \det(T^* - \bar{\lambda} \mathbb{1}) = 0$$

$$\Leftrightarrow \bar{\lambda} \in \sigma(T^*).$$