

608

i)

$$\begin{pmatrix} 0 & 1 & -2 & -4 \\ -1 & 0 & 2 & 1 \\ 2 & -2 & 0 & -3 \\ 4 & -1 & 3 & 0 \end{pmatrix} \xrightarrow{(-1)} \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -4 \\ 2 & -2 & 0 & -3 \\ 4 & -1 & 3 & 0 \end{pmatrix}$$

$$\xrightarrow{\sim} \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -4 \\ 0 & -2 & 4 & -1 \\ 0 & -1 & 1 & 4 \end{pmatrix} \xrightarrow{\sim} \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 0 & -9 \\ 0 & 0 & 9 & 8 \end{pmatrix}$$

$$\xrightarrow{(-1)} \begin{pmatrix} -1 & 0 & 2 & 1 \\ 0 & 1 & -2 & -4 \\ 0 & 0 & 9 & 8 \\ 0 & 0 & 0 & -9 \end{pmatrix}$$

$$\Rightarrow \det A = (-1) \cdot (-1) \cdot (-1) \cdot 9 / (-9) = \underline{\underline{81}}$$

c) $|A^T| = |A| = 81$, $|A^{-1}| = \frac{1}{|A|} = \frac{1}{81}$, $|A^2| = |A|^2 = 81^2$, $|A \cdot A| = |A| \cdot |A| = 81^2$,
 $A + A^T = 0 \Rightarrow |A + A^T| = 0$

d)

$$\det(A - tI) = \begin{vmatrix} -t & 1 & -2 & -4 \\ -1 & -t & 2 & 1 \\ 2 & -2 & -t & -3 \\ 4 & -1 & 3 & -t \end{vmatrix} \xrightarrow{\substack{1 \cdot 2 \quad 1 \cdot 4 \quad 1 \cdot (-t)}}} \begin{vmatrix} 0 & 1+t^2 & -2-2t & -4-t \\ -1 & -t & 2 & 1 \\ 0 & -2-2t & 4-t & -1 \\ 0 & -4-t & 11 & 4-t \end{vmatrix}$$

$$= \begin{vmatrix} 1+t^2 & -2-2t & -4-t & 1+t^2 & -2-2t \\ -2-2t & 4-t & -1 & -2-2t & 4-t \\ -4-t & 11 & 4-t & -4-t & 11 \end{vmatrix} = (1+t^2)(4-t)^2 - (2+2t)(4+t) + 11(2+2t)(4+t) - (4-t)(4+t)^2 + 11(1+t)^2 + (2+2t)^2(4-t)$$

$$e \dots = t^4 + 35t^2 + 81 \Rightarrow b = 35, a = 0 \quad (\text{Spur } A = 0)$$

$A - tI$ nicht invertierbar $\Leftrightarrow \det(A - tI) = 0 \Leftrightarrow (t^2)^2 + 35t^2 + 81 = 0$
 $\Leftrightarrow (t^2)_{1/2} = -35 \pm \sqrt{35^2 - 81} < 0 \Rightarrow \det(A - tI) \neq 0$ für alle $t \in \mathbb{R}$
 $\Rightarrow A - tI$ invertierbar für alle $t \in \mathbb{R}$.