A6

i) $S_2(e_1) = -e_1$, $S_2(e_2) = e_2$ ① + ①

$\Rightarrow M_{S_2} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$. ①

$D(e_1) = \frac{1}{\sqrt{2}} e_1 + \frac{1}{\sqrt{2}} e_2$, $D(e_2) = -\frac{1}{\sqrt{2}} e_1 + \frac{1}{\sqrt{2}} e_2$ ① + ①

$\Rightarrow M_D = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$. ①

ii)

$M_{D \circ S_2} = M_D \cdot M_{S_2} = \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} \cdot \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1/\sqrt{2} & -1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$ ②

$M_{S_2 \circ D} = M_{S_2} \cdot M_D = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$. ②

$\sum_{A6} = 10$

A7

$\ker f = \{ x \in \mathbb{R}^2 : x_1 + x_2 = 0 \} = \{ \lambda \begin{pmatrix} 1 \\ -1 \end{pmatrix} : \lambda \in \mathbb{R} \}$ ①

$\text{Bild } f = \{ f(x) : x \in \mathbb{R}^2 \} = \{ \begin{pmatrix} x_1 + x_2 \\ x_1 + x_2 \end{pmatrix} : x_1, x_2 \in \mathbb{R} \}$

$= \{ (x_1 + x_2) \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} : x_1, x_2 \in \mathbb{R} \} = \{ \lambda \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix} : \lambda \in \mathbb{R} \}$. ①

$\ker f \cap \text{Bild } f$:

$\lambda_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \lambda_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} \rightsquigarrow \lambda_1 \begin{pmatrix} 1 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} = 0 \Rightarrow \lambda_1 = \lambda_2 = 0$

$\rightarrow \ker f \cap \text{Bild } f = \{ 0 \}$ ①