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i) Wähle $e_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ und $e_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$.

$$e_1' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \quad e_2' = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$$

$$\|e_1'\| = 1, \quad \|e_2'\| = 1, \quad \langle e_1, e_1' \rangle = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right), \quad \langle e_2, e_1' \rangle = -\frac{1}{\sqrt{2}} = \cos\left(\frac{3\pi}{4}\right)$$

$$\langle e_2, e_2' \rangle = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right), \quad \langle e_1, e_2' \rangle = \frac{1}{\sqrt{2}} = \cos\left(\frac{\pi}{4}\right)$$

\Rightarrow 45° -Drehung gegen Uhrz. um den Nullpunkt.

Sei $b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $b_2 = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} b_1 = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}, \quad \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} b_2 = \begin{pmatrix} 0 \\ \sqrt{2} \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} \sqrt{2} & \sqrt{2} \\ 0 & \sqrt{2} \end{pmatrix} \text{ bzgl. der Basis } b_1, b_2.$$

ii) $e_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $e_2 = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$, $e_3 = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$.

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

Diese Matrix vertauscht (permutiert) die Koordinaten zyklisch.

1. \rightarrow 3., 2. \rightarrow 1., 3. \rightarrow 2.

$$b_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad b_2 = \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix}, \quad b_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}, \quad \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix} \text{ bzgl. der Basis } b_1, b_2, b_3$$

wähle nun linear unabhängige Teilmenge von

$$\{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \}. \quad \text{z.B. } \{ \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \end{pmatrix} \}. \quad \textcircled{1}$$