H22

i)

(SP1) $\hat{x}, \hat{y}, \hat{z} \in V$, $\lambda, \mu \in \mathbb{R}$.

$$\langle \lambda \hat{x} + \mu \hat{y}, \hat{z} \rangle = \sum_{i=1}^{\infty} (\lambda x_i + \mu y_i) z_i = \sum_{i=1}^{\infty} \lambda x_i z_i + \mu y_i z_i$$

$$= \sum_{i=1}^{\infty} \lambda x_i z_i + \sum_{i=1}^{\infty} \mu y_i z_i = \lambda \sum_{i=1}^{\infty} x_i z_i + \mu \sum_{i=1}^{\infty} y_i z_i$$

1

$$= \lambda \langle \hat{x}, \hat{z} \rangle + \mu \langle \hat{y}, \hat{z} \rangle$$

(SP2) $\langle \hat{x}, \hat{y} \rangle = \sum_{i=1}^{\infty} x_i y_i = \sum_{i=1}^{\infty} y_i x_i = \langle y, x \rangle$ 1/2

(SP3) $\langle \hat{x}, \hat{x} \rangle = \sum_{i=1}^{\infty} x_i^2 \geq 0$

Sei $\langle \hat{x}, \hat{x} \rangle = 0 = \sum_{i} x_i^2 \Rightarrow x_i^2 = 0 \Rightarrow x_i = 0 \Rightarrow \hat{x} = 0$ 1

$\Rightarrow$ Skalarprodukt! ✓

ii)

(SP1) $P_1, P_2, P_3 \in \mathbb{R}[x]$, $\lambda, \mu \in \mathbb{R}$ mit $P_1 = \sum_{i=0}^{m} a_i x^i$, $P_2 = \sum_{i=0}^{n} b_i x^i$, $P_3 = \sum_{i=0}^{l} c_i x^i$

$$\langle \lambda P_1 + \mu P_2, P_3 \rangle = \sum_{i \geq 0} (\lambda a_i + \mu b_i) c_i$$

$$= \sum_{i \geq 0} \lambda a_i \cdot c_i + \mu b_i c_i$$

$$= \sum_{i \geq 0} \lambda a_i \cdot c_i + \sum_{i \geq 0} \mu b_i c_i$$

1/2

$$= \lambda \sum_{i \geq 0} a_i \cdot c_i + \mu \sum b_i c_i = \lambda \langle P_1, P_3 \rangle + \mu \langle P_2, P_3 \rangle$$

(SP2)

$$\langle p_1, p_2 \rangle = \sum_{i\geq 0} a_i b_i = \sum_{i\geq 0} b_i a_i = \langle p_2, p_1 \rangle$$ 1/2

(SP3)

$$\langle p_1, p_1 \rangle = \sum_{i\geq 0} a_i^2 \geq 0$$

Sei $\langle p_1, p_1 \rangle = 0 \Rightarrow \sum_{i\geq 0} a_i^2 = 0 \Rightarrow a_i^2 = 0 \Rightarrow a_i = 0$ 1

$\Rightarrow p_1 = 0$

$\Rightarrow$ Skalarprodukt ✓

iii)

(SP1) $x, y, z \in \mathbb{C}^n$, $\lambda, \mu \in \mathbb{C}$.

$$\langle \lambda x + \mu y, z \rangle = \sum_{i=1}^{n} (\lambda x_i + \mu y_i) z_i = \sum_{i=1}^{n} \lambda x_i z_i + \mu y_i z_i$$

$$= \sum_{i=1}^{n} \lambda x_i z_i + \sum_{i=1}^{n} \mu y_i z_i = \lambda \sum_{i=1}^{n} x_i z_i + \mu \sum_{i=1}^{n} y_i z_i$$

$$= \lambda \langle x, z \rangle + \mu \langle y, z \rangle$$ 1

(SP2)

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i = \sum_{i=1}^{n} y_i x_i = \langle y, x \rangle$$ 1/2

(SP3)

$\langle x, x \rangle = \sum_{j=1}^{n} x_j^2 \overset{?}{=}$ denn $x_j^2 < 0$ ist möglich, siehe $i^2 = -1$!

zb. $x = \begin{pmatrix} i \\ \vdots \\ i \end{pmatrix} \Rightarrow \langle x, x \rangle = \sum_{j=1}^{n} i^2 = \sum_{j=1}^{n} -1 = -n.$ ↯

$\Rightarrow$ kein Skalarprodukt. 1

$\Sigma_{H22} = 7$