G23

$A\cdot A$, $B\cdot B$, $D\cdot D$, $B\cdot D$, $B\cdot C$, $D\cdot C$, $C\cdot A$, $D\cdot A$, $D\cdot B$ nicht möglich

$$A\cdot B = (1\ 3\ 5)\cdot\begin{pmatrix}2\\-1\\8\end{pmatrix} = 39 \quad , \quad B\cdot A = \begin{pmatrix}2\\-1\\8\end{pmatrix}(1\ 3\ 5) = \begin{pmatrix}2 & 6 & 10\\-1 & -3 & -5\\8 & 24 & 40\end{pmatrix}$$

$$A\cdot C = (1\ 3\ 5)\begin{pmatrix}3 & 0 & 1\\-1 & 1 & 8\\0 & 1 & 1\end{pmatrix} = (0\ \ 8\ \ 30)$$

$$C\cdot B = \begin{pmatrix}3 & 0 & 1\\-1 & 1 & 8\\0 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}2\\-1\\8\end{pmatrix} = \begin{pmatrix}14\\61\\7\end{pmatrix}$$

$$A\cdot D = (1\ \ 3\ \ 5)\begin{pmatrix}1 & 0 & -1 & 3 & 2\\2 & 0 & 1 & 0 & 4\\-1 & 2 & 5 & 0 & 1\end{pmatrix} = (2\ \ 10\ \ 27\ \ 3\ \ 19)$$

$$C\cdot D = \begin{pmatrix}3 & 0 & 1\\-1 & 1 & 8\\0 & 1 & 1\end{pmatrix}\cdot\begin{pmatrix}1 & 0 & -1 & 3 & 2\\2 & 0 & 1 & 0 & 4\\-1 & 2 & 5 & 0 & 1\end{pmatrix} = \begin{pmatrix}2 & 2 & 2 & 9 & 7\\-7 & 16 & 42 & -3 & 10\\1 & 2 & 6 & 0 & 5\end{pmatrix}$$

$$C\cdot C = \begin{pmatrix}3 & 0 & 1\\-1 & 1 & 8\\0 & 1 & 1\end{pmatrix}\begin{pmatrix}3 & 0 & 1\\-1 & 1 & 8\\0 & 1 & 1\end{pmatrix} = \begin{pmatrix}9 & 1 & 4\\-4 & 9 & 15\\-1 & 2 & 9\end{pmatrix}$$

G24

i) $S_1^2 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\cdot\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \mathbb{1}$, $S_2^2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}\cdot\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \mathbb{1}$

$S_3^2 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} = \begin{pmatrix}1 & 0\\0 & 1\end{pmatrix} = \mathbb{1}$

ii)

$$S_1S_2 + S_2S_1 = \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\cdot\begin{pmatrix}0 & -i\\i & 0\end{pmatrix} + \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}\cdot\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}$$
$$= \begin{pmatrix}i & 0\\0 & -i\end{pmatrix} + \begin{pmatrix}-i & 0\\0 & i\end{pmatrix} = 0$$

$$S_2S_3 + S_3S_2 = \begin{pmatrix}0 & -i\\i & 0\end{pmatrix}\cdot\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix} + \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & -i\\i & 0\end{pmatrix}$$
$$= \begin{pmatrix}0 & i\\i & 0\end{pmatrix} + \begin{pmatrix}0 & -i\\-i & 0\end{pmatrix} = 0$$

$$S_3S_1 + S_1S_3 = \begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}\begin{pmatrix}0 & 1\\1 & 0\end{pmatrix} + \begin{pmatrix}0 & 1\\1 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\0 & -1\end{pmatrix}$$
$$= \begin{pmatrix}0 & 1\\-1 & 0\end{pmatrix} + \begin{pmatrix}0 & -1\\1 & 0\end{pmatrix} = 0$$

iii)

$$S_1 S_2 - S_2 S_1 = \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix} - \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$$

$$= \begin{pmatrix}i&0\\0&-i\end{pmatrix} - \begin{pmatrix}-i&0\\0&i\end{pmatrix} = 2i\begin{pmatrix}1&0\\0&-1\end{pmatrix} = 2i\, S_3$$

$$S_3 S_1 - S_1 S_3 = \begin{pmatrix}1&0\\0&-1\end{pmatrix}\cdot\begin{pmatrix}0&1\\1&0\end{pmatrix} - \begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$$

$$= \begin{pmatrix}0&1\\-1&0\end{pmatrix} - \begin{pmatrix}0&-1\\1&0\end{pmatrix} = 2i\begin{pmatrix}0&-i\\i&0\end{pmatrix} = 2i\, S_2$$

$$S_2 S_3 - S_3 S_2 = \begin{pmatrix}0&-i\\i&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix} - \begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&-i\\i&0\end{pmatrix}$$

$$= \begin{pmatrix}0&i\\i&0\end{pmatrix} - \begin{pmatrix}0&-i\\-i&0\end{pmatrix} = 2i\begin{pmatrix}0&1\\1&0\end{pmatrix} = 2i S_1 .$$

<u>G25</u>

i) $\begin{pmatrix}1&1\\0&1\end{pmatrix}\cdot\begin{pmatrix}a&b\\c&d\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix} \rightsquigarrow$ LGS

$a + c = 1$, $b + d = 0$, $c = 0$, $d = 1$.

$\Rightarrow a = 1$, $b = -1$. $\Rightarrow \begin{pmatrix}1&1\\0&1\end{pmatrix}^{-1} = \begin{pmatrix}1&-1\\0&1\end{pmatrix}$

ii) nicht möglich,

Rang $\begin{pmatrix}1&1\\0&0\end{pmatrix}$ = dim Bild$\begin{pmatrix}1&1\\0&0\end{pmatrix} = 1$

dim ker $\begin{pmatrix}1&1\\0&0\end{pmatrix} = 1 \Rightarrow \begin{pmatrix}1&1\\0&0\end{pmatrix}$ nicht injektiv.

iii)

$$\begin{pmatrix}1&1&2\\1&0&1\end{pmatrix}\begin{pmatrix}a&b\\c&d\\e&f\end{pmatrix} = \begin{pmatrix}1&0\\0&1\end{pmatrix}$$

$a + c + 2e = 1$, $b + d + 2f = 0$

$a + e = 0$, $b + f = 1$; 2 frei wählbare Parameter $a = r$, $b = s$

$\Rightarrow e = -r$, $f = 1 - s$

$c = 1 + r$; $d = -2 + s$

$$\mathbb{L} = \left\{ r \cdot \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ -1 & 0 \end{pmatrix} + s \cdot \begin{pmatrix} 0 & 1 \\ 0 & 1 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & -2 \\ 0 & 1 \end{pmatrix} \right\}$$

wähle für $r = s = 0$ $\quad \begin{pmatrix} 1 & 1 & 2 \\ 1 & 0 & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 0 & 0 \\ 1 & -2 \\ 0 & 1 \end{pmatrix}$.