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$$i) \left\langle \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix} \right\rangle = \cos^2 \theta + \sin^2 \theta = 1$$

$$\left\langle \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix}, \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} \right\rangle = \sin^2 \theta + \cos^2 \theta = 1$$

$$\left\langle \begin{pmatrix} \cos \theta \\ \sin \theta \end{pmatrix}, \begin{pmatrix} -\sin \theta \\ \cos \theta \end{pmatrix} \right\rangle = -\cos \theta \sin \theta + \sin \theta \cos \theta = 0 \quad \Rightarrow \underline{\underline{ONB}} \quad \checkmark$$

ii)

$$\left\langle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle = \frac{1}{2} + 0 + \frac{1}{2} = 1$$

$$\left\langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle = 1, \quad \left\langle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle = \frac{1}{2} + 0 - \frac{1}{2} = 0$$

$$\left\langle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix} \right\rangle = \frac{1}{2} - \frac{1}{2} = 0$$

$$\left\langle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle = 0, \quad \left\langle \begin{pmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \right\rangle = 0 \quad \Rightarrow \underline{\underline{ONB}} \quad \checkmark$$