$$Q(x_1, x_2) = \lambda_1 x_2^2 + \lambda_2 x_2^2$$

$\lambda_1, \lambda_2 > 0$

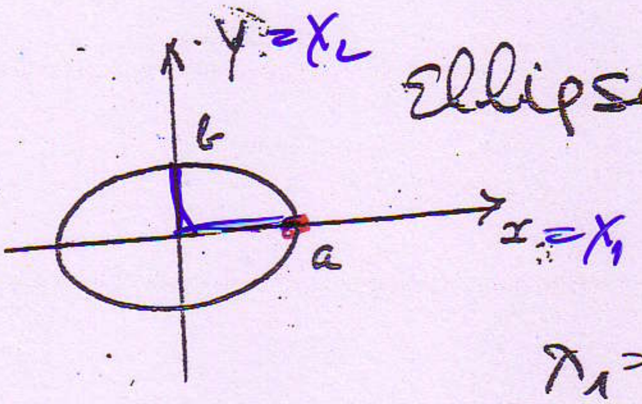
Kennlinie $n=2$

$x_2 = 0$

$\lambda_1 x_1^2 = 1$

$x_1 = \pm \frac{1}{\sqrt{\lambda_1}}$

$a = \frac{1}{\sqrt{\lambda_1}}$

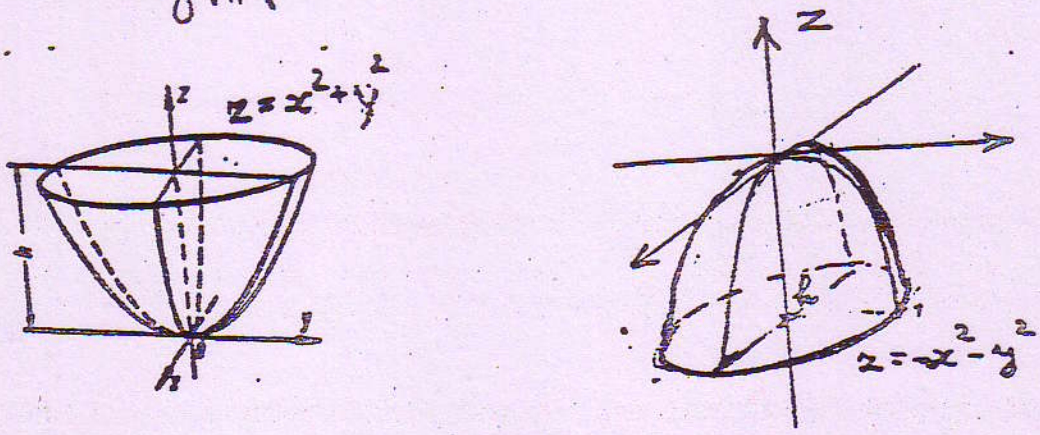


Ellipse

$\lambda_1 = \frac{1}{a^2}$

$\frac{1}{a^2} x^2 + \frac{1}{b^2} y^2 = 1$ $\quad \lambda_2 = \frac{1}{b^2}$

Graph $n=2$

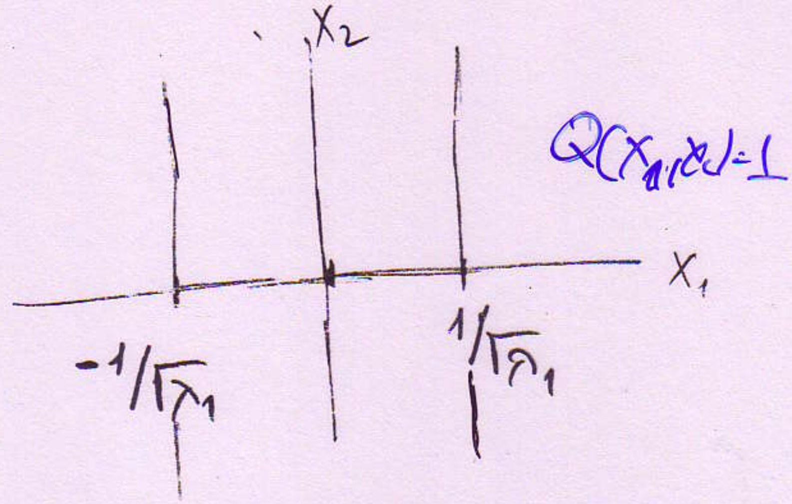


elliptische Paraboloide

positiv definit		negativ definit
$Q(x_1, x_2) > 0$	$(x_1, x_2) \neq (0,0)$	$Q(x_1, x_2) < 0$
Minimum		Maximum

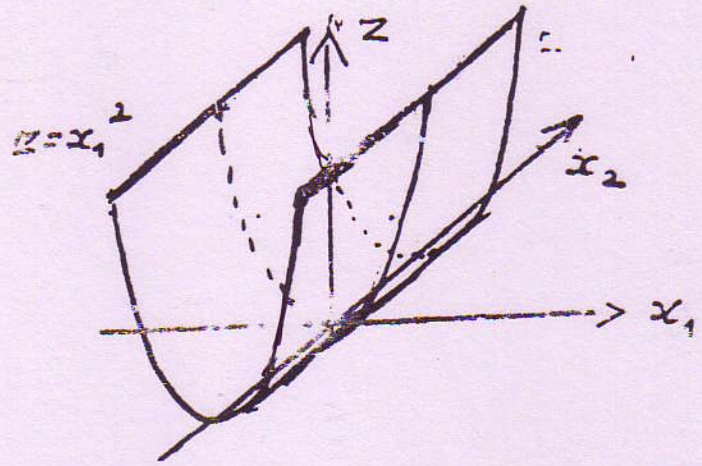
$Q(x_1, x_2) = \lambda_1 x_1^2 + 0 x_2^2 = 1$

$\lambda_1 > 0$



Kennlinie: Geradenpaar

Fläche



positiv semidefinit $Q(x_1, x_2) \geq 0$

parabolischer Zylinder

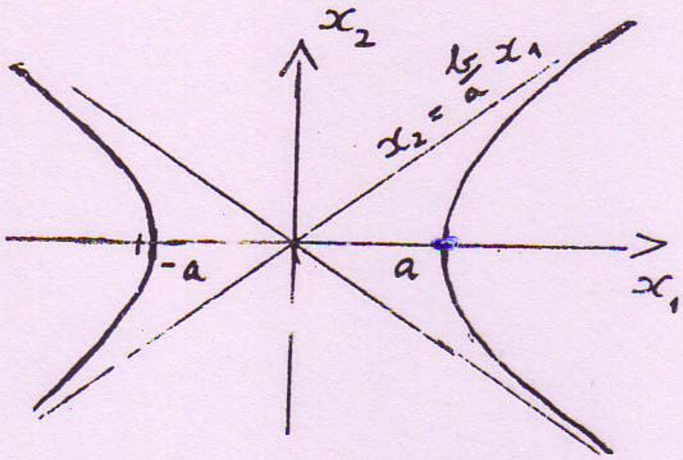
$$Q(x_1, x_2) = \lambda_1 x_1^2 + \lambda_2 x_2^2 = 1$$

$\lambda_1 > 0 \,,\; \lambda_2 < 0$

$$a = \frac{1}{\sqrt{\lambda_1}} \quad b = \frac{1}{\sqrt{-\lambda_2}}$$

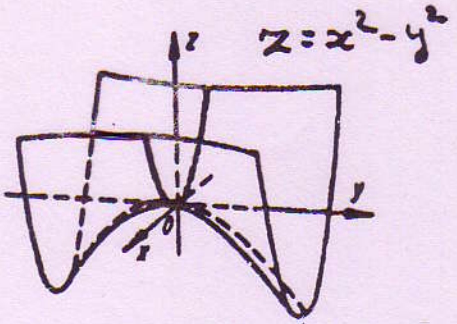
$$\frac{x_1^2}{a^2} - \frac{x_2^2}{b^2} = 1 \approx 0$$

$$\frac{x_2}{x_1} = \pm \frac{b}{a}$$



Kennlinie Hyperbel

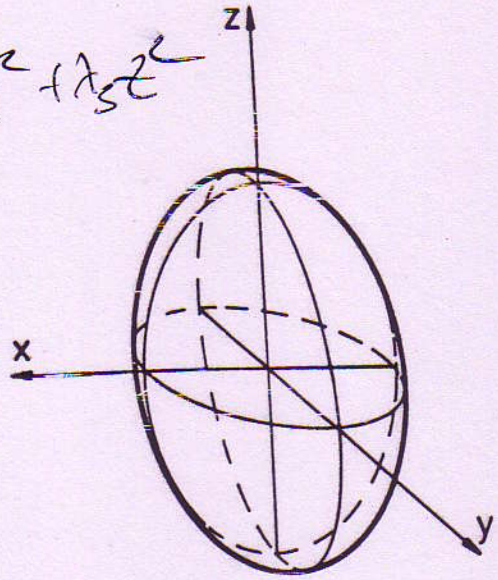
Fläche:
hyperbolisches
Paraboloid
Sattelfläche
indefinit



27.1.2

$Q(\vec{x}) = \lambda_1 x^2 + \lambda_2 y^2 + \lambda_3 z^2$

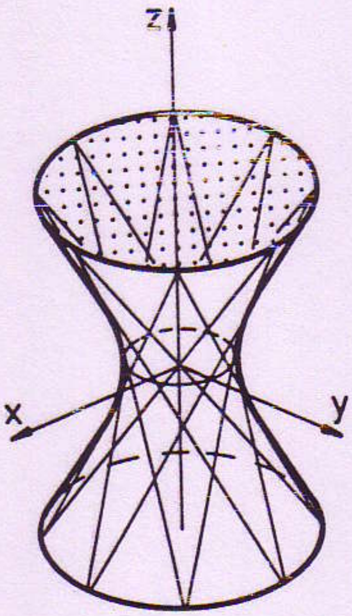
$Q(\vec{x}) = 1$

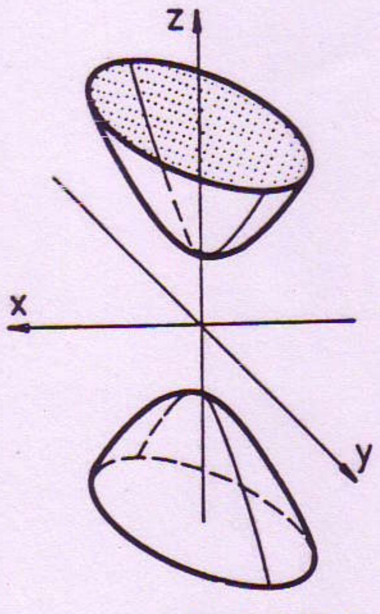


Ellipsoid

$\lambda_i > 0$

positiv definit





Einschaliges Hyperboloid

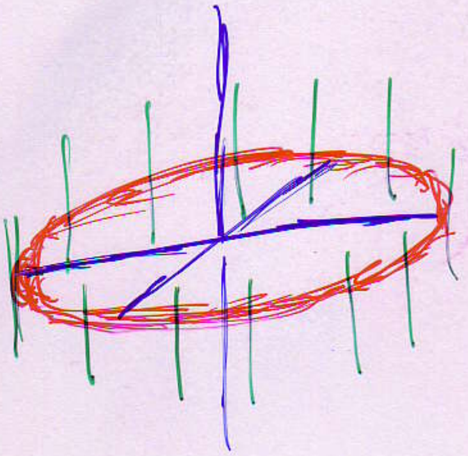
Zweischaliges Hyperboloid

$\lambda_1, \lambda_2 > 0$, $\lambda_3 < 0$

$\lambda_1, \lambda_2 < 0$, $\lambda_3 > 0$

indefinit

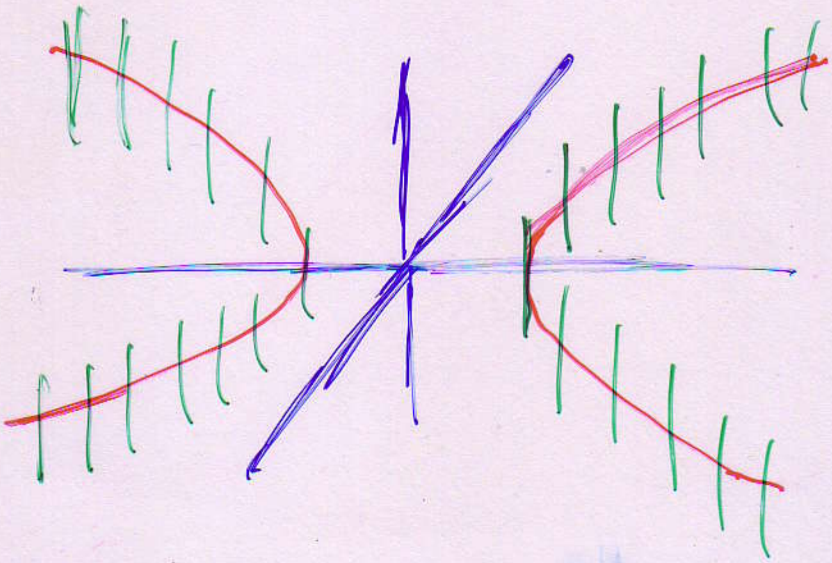
$$\lambda_1 x_1^2 + \lambda_2 x_2^2 + \lambda_3 x_3^2 = 1$$



$\lambda_1 x_1^2 + \lambda_2 x_2^2 + 0\, x_3^2 = 1$

$\lambda_1, \lambda_2 > 0$

elliptischer Zylinder



$\lambda_1 x^2 + \lambda_2 x_2^2 + 0\, x_3^2 = 1$

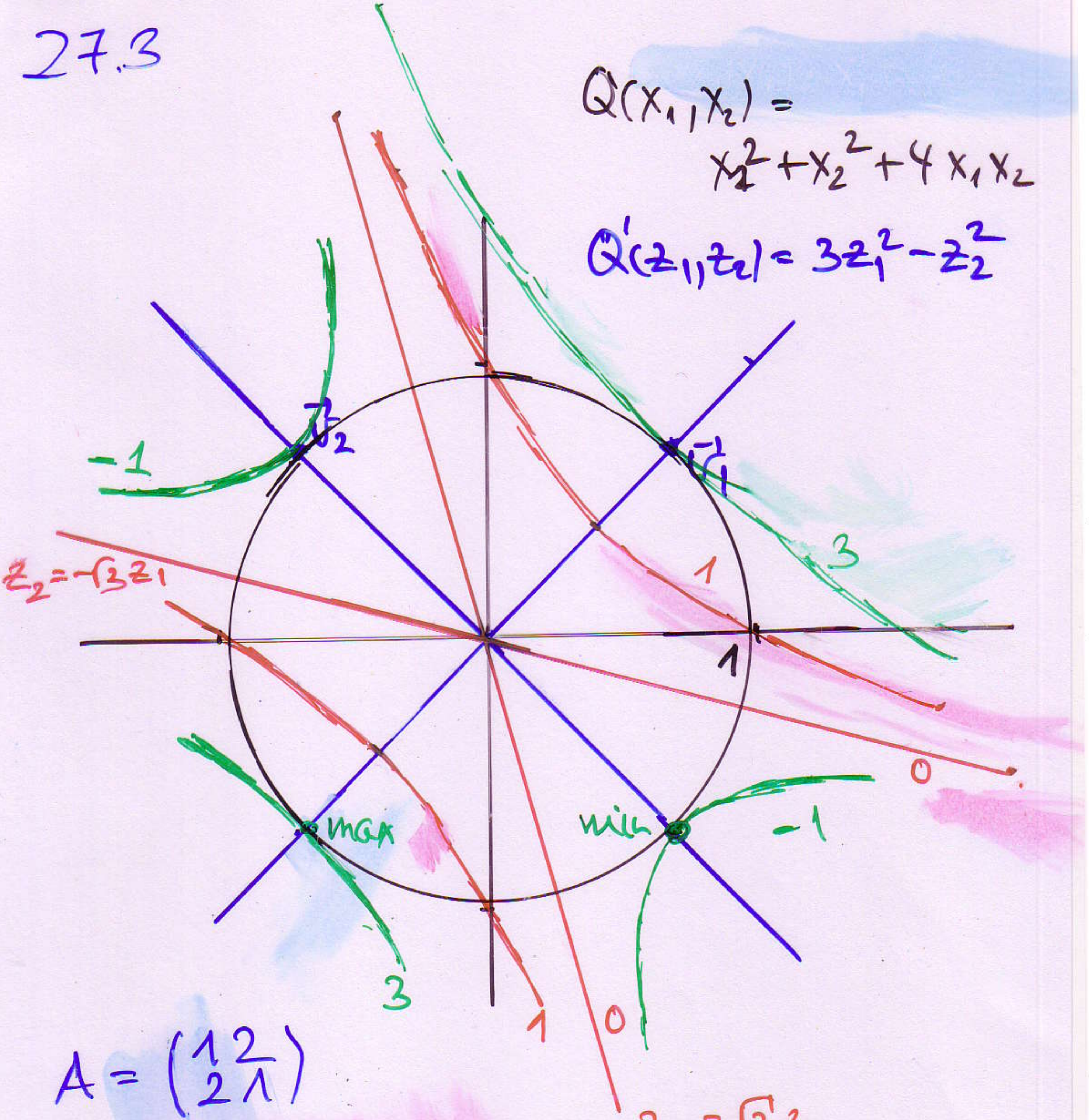
$\lambda_1 > 0 > \lambda_2$

hyperbolischer Zylinder

27.3

$Q(x_1, x_2) = x_1^2 + x_2^2 + 4x_1x_2$

$Q'(z_1, z_2) = 3z_1^2 - z_2^2$



$A = \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

Ansatz

$(1, t)\, A \begin{pmatrix} -t \\ 1 \end{pmatrix} = 0$

$= 2(1 - t^2)$

$A' = S^t A S = \begin{pmatrix} 3 & 0 \\ 0 & -1 \end{pmatrix}$

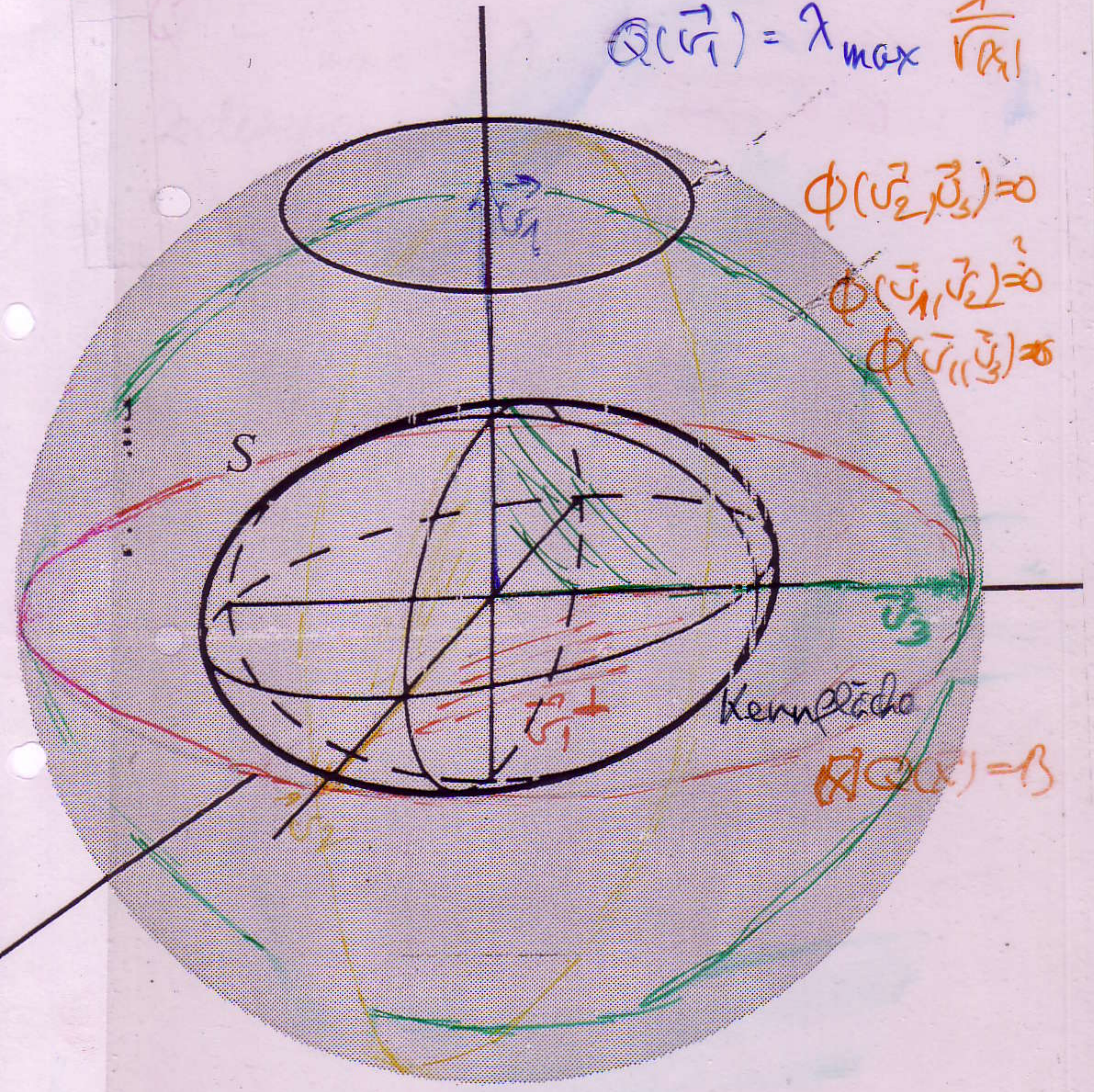
$t = \pm 1 \quad \vec{v}_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \vec{v}_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} -1 \\ 1 \end{pmatrix}$

$\lambda_1 = Q(\vec{v}_1) = 3 \qquad \lambda_2 = Q(\vec{v}_2) = -1$

$Q(\vec{x}) = \lambda_1 x_1^2 + \dots + \lambda_n x_n^2 \quad \lambda_1 \geq \lambda_2 \geq \dots$

$x_2 = 0$

$Q(\vec{v}_1) = \lambda_{max} \quad \frac{1}{\sqrt{|\alpha_1|}}$



Einheitskugel $|\vec{x}| = 1$