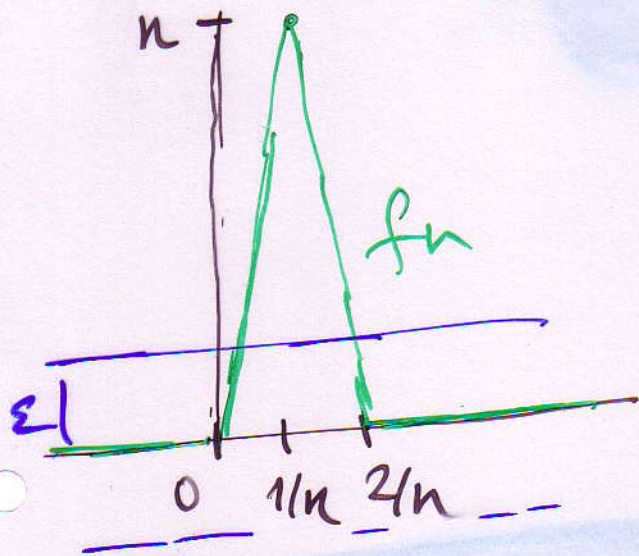
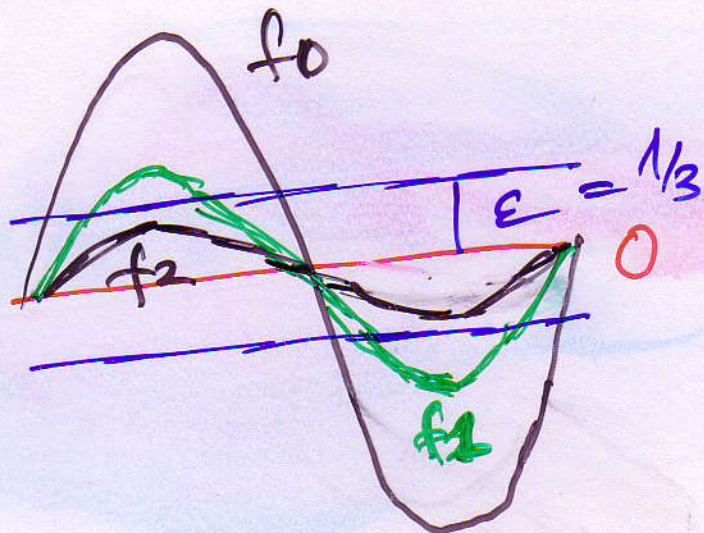


25.2 Gleichmäßige Konvergenz

$$\|f\|_\infty = \sup_{x \in D} f(x) \quad \text{Supremumsnorm}$$

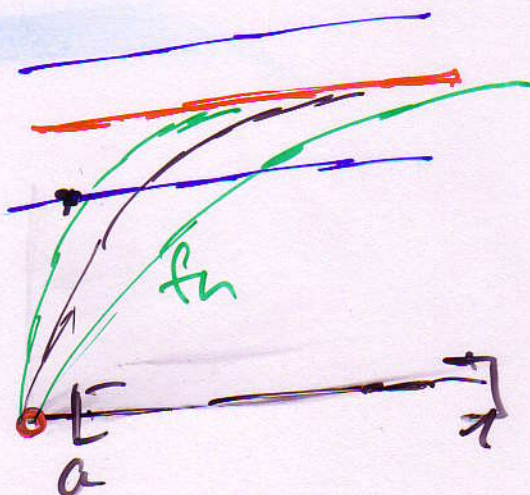
$$f_n \rightarrow f \text{ glm} \Leftrightarrow \|f_n - f\|_\infty \rightarrow 0$$

$$\frac{\sin nx}{2^n} \rightarrow 0 \text{ glm}$$



$f_n \rightarrow 0$
nicht glm

$$f_n(x) = \frac{1}{1+nx} \quad x \in [0, 1]$$



$$\|f_n - f\|_\infty = 1$$

$\rightarrow f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \end{cases}$
nicht glm

glm auf $[a, 1]$ $a > 0$

Lemma 25.6 $\exists f, f_n \rightarrow f$ gdw

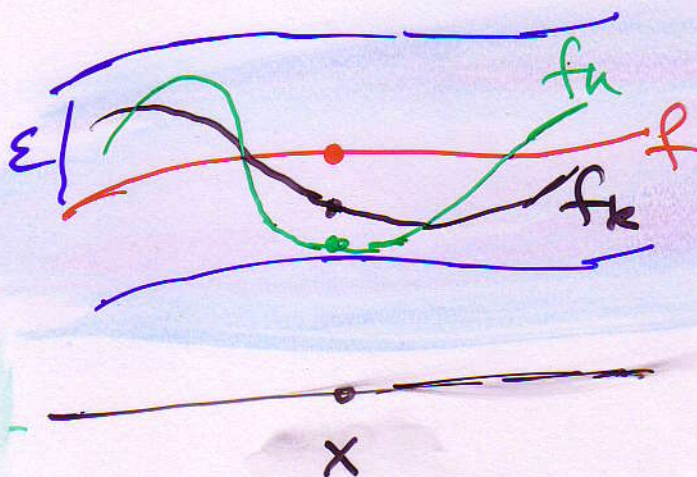
$\Leftrightarrow \forall \varepsilon > 0 \exists n_\varepsilon \forall m, n \geq n_\varepsilon$

$$\|f_n - f_m\|_\infty \leq \varepsilon$$

Bew. x fest $f_n(x)$ Cauchy

$\Rightarrow \exists f(x) \rightarrow f(x)$

$\varepsilon > 0 \quad n_{\varepsilon/2}$



$n \geq n_{\varepsilon/2} \quad \|f - f_n\|_\infty \leq \varepsilon ?$

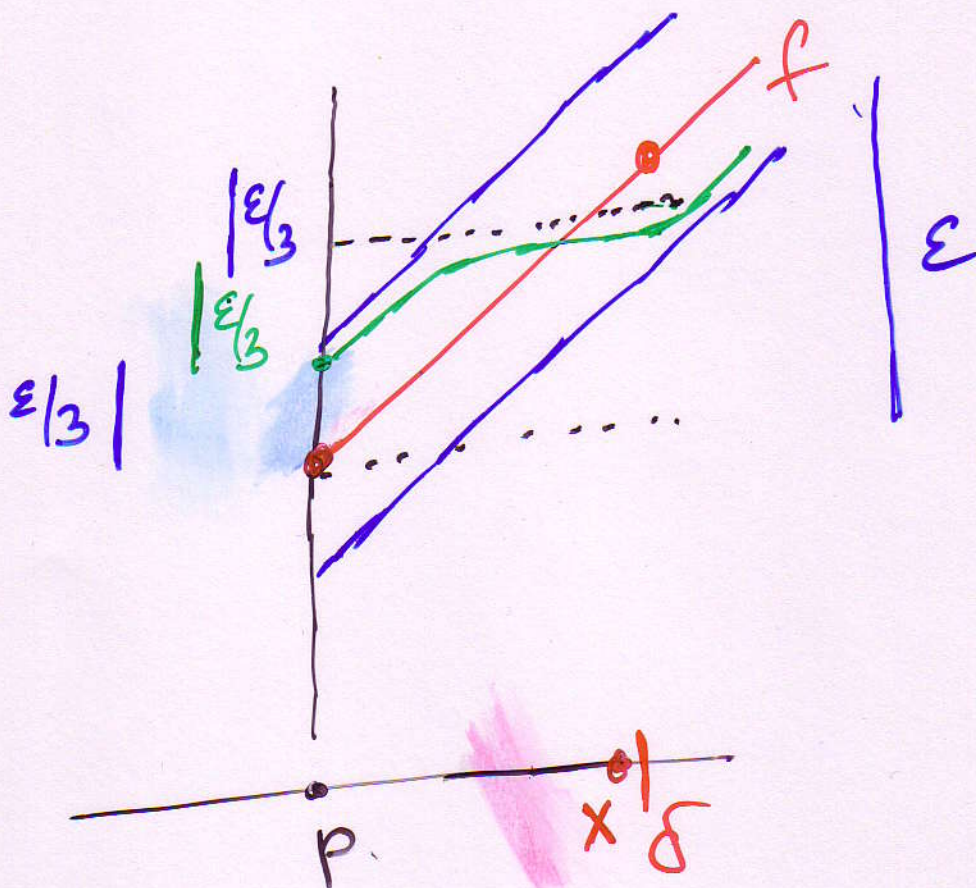
$x \in D \quad \exists k \geq n_{\varepsilon/2} \quad |f_k(x) - f(x)| \leq \varepsilon/2$

$$|f(x) - f_n(x)| =$$

$$\leq |f(x) - f_k(x)| + |f_k(x) - f_n(x)|$$

$$\leq \varepsilon/2 + \varepsilon/2 = \varepsilon$$

Satz 25.7 $f_n \rightarrow f$ glm
 f_n stetig $\Rightarrow f$ stetig



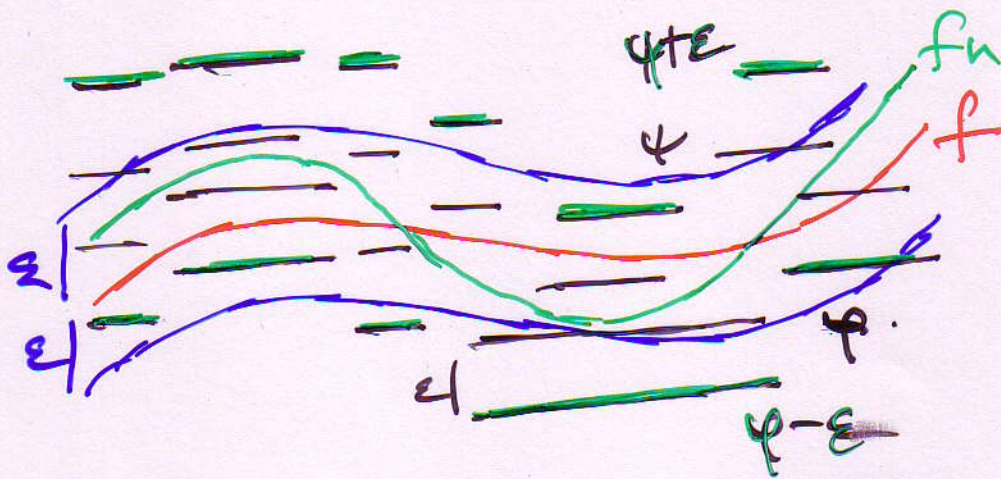
$$\|f_n - f\| \leq \epsilon/3$$

$$\forall x \quad |x - p| < \delta \Rightarrow |f_n(x) - f_n(p)| \leq \epsilon/3$$

$$\rightarrow |f(p) - f(x)|$$

$$\leq |f(p) - f_n(p)| + |f_n(p) - f_n(x)| + |f_n(x) - f(x)|$$

$$\leq \epsilon/3 + \epsilon/3 + \epsilon/3$$



Satz
25.13

$f_n: [a, b] \rightarrow \mathbb{R}$, $f_n \rightarrow f$ glm
integrierbar

$$\Rightarrow \int_a^b f = \lim_{n \rightarrow \infty} \int_a^b f_n$$

Bew. $\epsilon > 0 \leadsto \|f_n - f\|_{\infty} \leq \epsilon$ für ein n

$\varphi \leq f_n \leq \psi$ Treppenfkt $\int \psi - \varphi \leq \epsilon$

$$\varphi - \epsilon \leq f \leq \psi + \epsilon$$

$$\int \psi + \epsilon - (\varphi - \epsilon) \leq \epsilon(2(b-a) + 1) \xrightarrow{\text{für } \epsilon \rightarrow 0} 0$$

$\Rightarrow f$ integrierbar

$$|\int f_n - \int f| = |\int f_n - f| \leq \int |f_n - f|$$

$$\leq \int \epsilon = \epsilon(b-a) \rightarrow 0$$