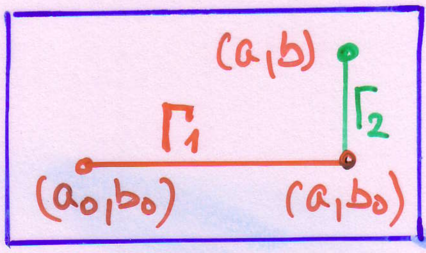
20.3.3 Beweis



$$\varphi(a,b) = \int_{\Gamma_1} F \cdot d\vec{x} + \int_{\Gamma_2} F \cdot d\vec{x}$$

$$= \int_{a_0}^{a} \langle F(t,b_0) \,|\, \vec{e}_1 \rangle \, dt + \int_{b_0}^{b} \langle F(a,t) \,|\, \vec{e}_2 \rangle \, dt$$

$$= \int_{a_0}^{a} F_1(t,b_0)\, dt + \int_{b_0}^{b} F_2(a,t)\, dt$$

$$\frac{\partial \varphi}{\partial x}(a,b) \overset{?}{=} F_1(a,b)$$

Hauptsatz $\Rightarrow$ $\dfrac{\partial}{\partial x} \displaystyle\int_{a_0}^{a} F_1(t,b_0)\, dt = F_1(a,b_0)$

$$F_1(a,b) - F_1(a,b_0) = \int_{b_0}^{b} \underbrace{\frac{\partial F_1}{\partial y}}_{\| !}(a,t)\, dt$$

Vertauschung $\Rightarrow$

$$\frac{\partial}{\partial x} \int_{b_0}^{b} F_2(a,t)\, dt = \int_{b_0}^{b} \overbrace{\frac{\partial F_2}{\partial x}}(a,t)\, dt$$

$$\frac{\partial \varphi}{\partial y}(a,b) = \frac{\partial}{\partial y} \int_{b_0}^{b} F_2(a,t)\, dt = F_2(a,b)$$