Fachbereich Mathematik
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8.06.2009

Partielle Differentialgleichungen

8. Übung

Hausübung

H 1 Let $u(x,y) = \ln\sqrt{x^2+y^2}$ for $U = \mathbb{R}^2 \setminus \{(0,0)\}$.

1. Prove that $\Delta u(x,y) = 0$ for $(x,y) \in U$.
2. For $r > 0$ calculate the integral
$$\int_{\partial B(0,r)} \frac{\partial u}{\partial \nu}(y) dS_y.$$
3. Explain why the above result doesn't contradict Gauss's theorem.

H 2 Prove that $\Delta u = v_{rr} + \frac{1}{r}v_r + \frac{1}{r^2}v_{\phi\phi}$ in radial coordinates.
Hint: Let
$$x = r\cos\phi, \quad y = r\sin\phi.$$
Take $u(x,y) = v(r,\phi) = v\left(\sqrt{x^2+y^2}, \arctan\frac{y}{x}\right)$. Then compute $\partial_x u$, $\partial_{xx} u$, $\partial_y u$, $\partial_{yy} u$ using the chain rule.