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Partielle Differentialgleichungen

1. Übungen

Gruppenübung

G 1 The Burgers' equation

We will study the Burgers' equation

$$\begin{aligned} u_t(t,x) + u(t,x)u_x(t,x) &= 0 \quad \text{for } (t,x) \in \mathbb{R}_+ \times \mathbb{R} \\ u(0,x) &= u^0(x) \quad \text{for } x \in \mathbb{R}. \end{aligned} \tag{$\star$}$$

1. Define $v(s) = u(t(s), x(s))$ for $s \in \mathbb{R}_+$ and some differentiable functions $t(s)$ and $x(s)$. Compute the derivative of v.
2. Suppose that u is the solution to the initial problem ($\star$).
   (a) What are the conditions on $\dot{t}$ and $\dot{x}$ for v to be a constant function?
   (b) What is the value of this constant?
   (c) Write out the functions t and x explicitly.
3. Let
   (a) $u^0(x) = 1$,
   (b) $u^0(x) = -1$,
   (c)
   $$u^0(x) = \begin{cases} 1 & \text{for } x < 0 \\ 1 - x & \text{for } 0 \le x < 1 \\ 0 & \text{for } 1 \le x. \end{cases}$$
   Draw the curves $(x(s), t(s))$ for $t(0) = 0$ and different values of $x(0) \in \mathbb{R}$.
   (d) Let $u(t,x)$ be the solution of ($\star$) with initial condition (c). What is the value $u(1,1)$?

Hausübung

H 1 Non-homogeneous equations Proceeding similarly as above, find the solutions to the following problems:

1. $u_t + \alpha u_x = 1$,
2. $u_t + \alpha u_x = \beta u$,

where $\alpha, \beta \neq 0$ and $u(0,x) = u^0(x)$.