Äquivalent für $p \in D$, $\vec{a} \in V$

(1) $\vec{a} = f'(p) = \lim\limits_{\substack{\Delta t \to 0 \\ \neq 0}} \dfrac{\Delta \vec{y}}{\Delta t}$

(2) $\Delta \vec{y} = \vec{a}\,\Delta t + \varphi(\Delta t) \cdot \Delta t$

mit $\lim\limits_{\Delta t \to 0} \varphi(\Delta t) = 0$

$\varphi(\Delta t) = \dfrac{\Delta \vec{y}}{\Delta t} - \vec{a}$

(3) $\Delta \vec{y} = \vec{a}\,\Delta t + R(\Delta t)$

mit $\lim\limits_{\Delta t \to 0} \dfrac{R(\Delta t)}{\Delta t} = 0$

$R(\Delta t) = \varphi(\Delta t) \cdot \Delta t$

$$f: D \to V, \quad f(t) = \vec{y}, \quad]p-\gamma, p+\gamma[\subseteq D$$

Differenz $\Delta\vec{y}(p): V \to V$

$$\Delta\vec{y}(p)(\Delta t) = f(p+\Delta t) - f(p)$$

Differentialquotient $\dfrac{\partial\vec{y}}{\partial t}(p) \in V$

$$\frac{\partial\vec{y}}{\partial t}(p) = \lim_{\substack{\Delta t \to 0 \\ \neq 0}} \frac{\Delta\vec{y}}{\Delta t} = f'(p) = \dot{\vec{y}}(p)$$

Differential $d\vec{y}(p): V \to V$

$$d\vec{y}(p)(dt) = \frac{\partial\vec{y}}{\partial t}(p) \cdot dt$$

$V = \mathbb{R}, \mathbb{C}$, Vektorraum

$$f(t) = f_1(t)\vec{e}_1 + \quad + f_n(t)\vec{e}_n$$

$$f'(p) = \vec{a} \iff f_k'(p) = a_k \qquad k = 1, \ldots, n$$

Bew. $\varphi(\Delta t) = \varphi_1(\Delta t)\vec{e}_1 + \ldots + \varphi_n(\Delta t)\vec{e}_n$

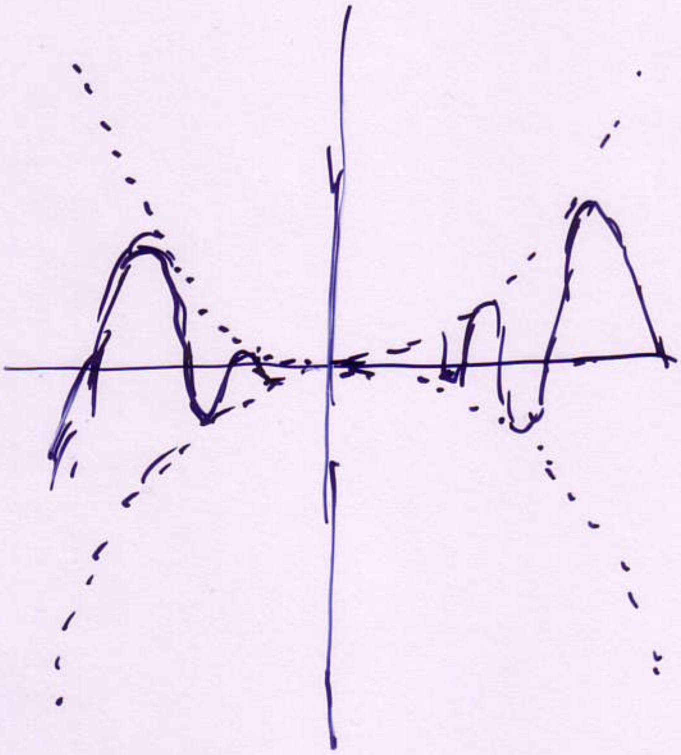
$$\Delta\vec{y} = \Delta y_1 \vec{e}_1 + \quad + \Delta y_n \vec{e}_n$$

$$= \vec{a}\,\Delta t + \varphi(\Delta t)\,\Delta t$$

$$\Delta y_k = a_k \Delta t + \varphi_k(\Delta t)\Delta t\,\vec{e}_k$$

$$\varphi(\Delta t) \to 0$$

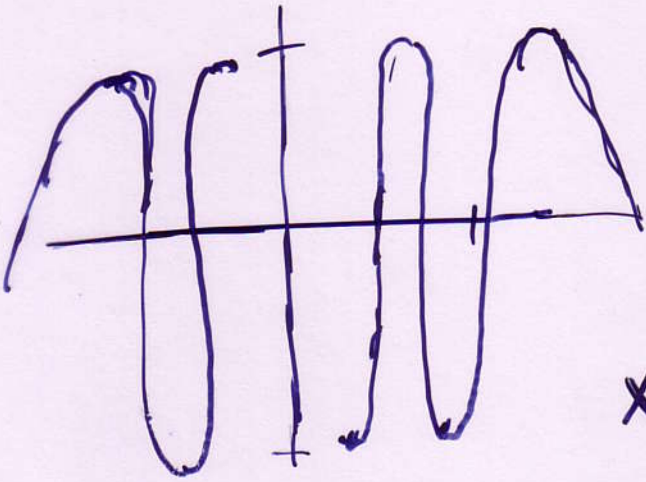
$$\iff \varphi_k(\Delta t) \to 0 \qquad k = 1, \ldots, n$$



$$y = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

differenzierbar an 0

$$\frac{\Delta y}{\Delta x} = \Delta x \sin\frac{1}{\Delta x} \to 0$$

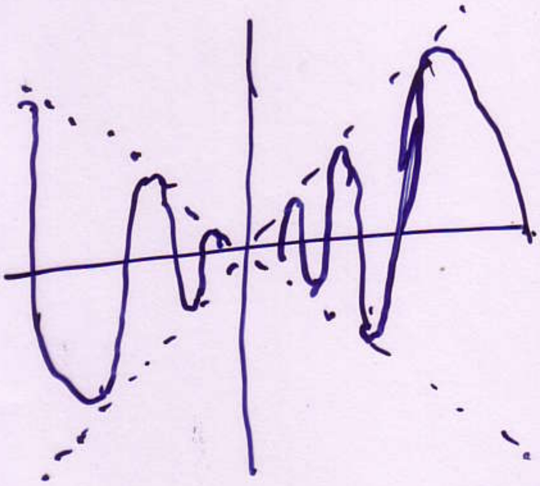


$$y = \begin{cases} \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

nicht stetig an 0

$$x_n = \frac{1}{(2n+\frac{1}{2})\pi} \quad f(x_n) \to 1$$

$$x_n = \frac{1}{(2n-\frac{1}{2})\pi} \quad f(x_n) \to -1$$



$$y = \begin{cases} x \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases}$$

stetig an 0
nicht differenzierbar

$$\Delta y = \Delta x \sin\frac{1}{\Delta x} \to 0$$

$$\frac{\Delta y}{\Delta x} = \sin\frac{1}{\Delta x} \begin{matrix} \nearrow 1 \\ \to -1 \end{matrix}$$