Transformationssatz für Integrale:

$$\int_{\Omega} h(X(\omega))\, dP(\omega) = \int_{\mathbb{R}} h(x)\, dP_X(x).$$

<u>Satz:</u>

$$\int_{\mathbb{R}} h(x)\, dP_X(x) = \begin{cases} \sum_{k=1}^{\infty} h(x_k) \cdot P[X = x_k], & \text{falls } P_X(\{x_1, x_2, \ldots\}) = 1 \\ \int_{-\infty}^{\infty} h(x) \cdot f(x)\, dx, & \text{falls } X \text{ Dichte } f \text{ hat} \end{cases}$$

<u>Korollar:</u>

$$E[h(X)] = \begin{cases} \sum_{k=1}^{\infty} h(x_k) \cdot P[X = x_k], & \text{falls } P_X(\{x_1, x_2, \ldots\}) = 1 \\ \int_{\mathbb{R}} h(x) \cdot f(x)\, dx, & \text{falls } X \text{ Dichte } f \text{ hat} \end{cases}$$

z.B. $X \sim b(n, p) \Rightarrow E(X^2) = \sum_{k=0}^{n} k^2 \cdot \binom{n}{k} \cdot p^k (1-p)^{n-k}$

z.B. $X \sim \mathcal{N}(\mu, \sigma^2)$

$$\Rightarrow E(e^X) = \int_{-\infty}^{\infty} e^x \cdot \frac{1}{\sqrt{2\pi}} \cdot \frac{1}{\sigma} \cdot e^{-\frac{(x-\mu)^2}{2\sigma^2}}\, dx$$