Fachbereich Mathematik
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7th exercise sheet Set Theory
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(E7.1)

Prove the inequality $\kappa < \kappa^{\mathrm{cf}(\kappa)}$ without using the Axiom of Choice.

Hint: Use a diagonalisation argument.

(E7.2)

If $\alpha < \kappa^+$, then there are subsets $X_n \subseteq \alpha$ $(n \in \mathbb{N})$ such that $\bigcup_{n\in\mathbb{N}} X_n = \alpha$ and $\overline{X_n} \leqslant_o \kappa^n$. (This is the so-called Milner-Rado Paradox.)

Hint: Use induction on α, and at limit stages consider a cofinal map $\beta \to \alpha$ with $\beta \leqslant \kappa$.

(E7.3)

Let κ be an infinite cardinal, well-ordered by $\leqslant$. If $\leqslant'$ is another well-order on κ, then there is a subset $X \subseteq \kappa$ such that: $|X| = \kappa$, and $\leqslant$ and $\leqslant'$ agree on X.

Hint: First prove the statement for regular cardinals κ. For singular cardinals, use a cofinal map $\mathrm{cf}(\kappa) \to \kappa$.