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Introduction to Mathematical Software

4^{th} Exercise Sheet

Exercise 1 (Matrices)

Consider the following matrices.

$$A = \begin{bmatrix} 2 & 3 & 1 \\ 2 & -4 & 3 \\ 4 & 5 & 9 \end{bmatrix}, \qquad B = \begin{bmatrix} -1 & 2 \\ 1 & 0 \\ 7 & 4 \end{bmatrix}$$

Compute

(a) A^{-1}
(b) AA^T
(c) B^TAB
(d) $(2A + BB^T)A^T$

Hint: You may want to have a look at the help of the package `LinearAlgebra`.

Exercise 2 (Determinants)

(a) Compute $\det \begin{bmatrix} x^2+1 & x & 0 & 0 \\ x & x^2+1 & x & 0 \\ 0 & x & x^2+1 & x \\ 0 & 0 & x & x^2+1 \end{bmatrix}$

(b) Compute $\det \begin{bmatrix} x^2+1 & x & 0 & 0 & 0 \\ x & x^2+1 & x & 0 & 0 \\ 0 & x & x^2+1 & x & 0 \\ 0 & 0 & x & x^2+1 & x \\ 0 & 0 & 0 & x & x^2+1 \end{bmatrix}$

(c) Looking at the results of (a) and (b), do you have any idea what the determinant of a general matrix of the above form is? If so, check your conjecture for an 8 x 8 matrix. If not, compute the determinants for matrices of dimension 6 and 7 to get an idea.

Exercise 3 (3-dimensional geometry)

Let a sphere S and a line g be given. S has its center at $(0, 0, 0)$ and a radius $r = 1$. g intersects the sphere at two points and goes through the point $(0, 0, 0)$. Produce a drawing of S, the line g intersecting the sphere, and a tangent plane such that line and plane are orthogonal to each other.
Helpful Maple-commands: `with(geom3d)`, `sphere`, `intersection`, `TangentPlane`, `line`, `draw`, `detail`.
Especially look at the intersection-object of line and sphere: `g_intersect1_S`.

Exercise 4 (Number theory)

For each natural number n, the $n \times n$ matrix A_n is defined as

$$A_n(i, j) = \gcd(i, j).$$

(a) Compute the determinant of A_n for $n = 1, 2, \ldots, 15$.

(b) Try to find a closed formula for the general case.
Hint: Find a connection to Euler's totient function (`phi,numtheory`)