May 11, 2006
Introduction to Compact Groups

We discussed projective limits of projective systems of compact groups. The important result was

Proposition A. *Let*

$$\{G_j, j \in J; f_{jk}\colon G_k \to G_j,\ j,k \in J, j \leq k\}$$

be a projective system of compact groups and let $G = \lim_{j\in J} G_j$ *be the limit with limit morphisms* $f_j\colon G \to G_j$. *Then the following statements are equivalent:*
(1) All f_j *are surjective,*
(2) All f_{jk} *are surjective.*

We shall call a projective system satisfying (2) a *strict projective system.*

Proposition B. *Let* G *be a compact group and* $\mathcal{N}$ *a filter basis of compact normal subgroups intersecting in* $\{1\}$. *Then*

$$\{G/N, N \in \mathcal{N}; f_{MN}\colon G/N \to G/M,\ M,N \in \mathcal{N}, N \subseteq M\},$$

$f_{MN}(gN) = gN$, *is a strict projective system and* $G \cong \lim N \in \mathcal{N} G/N$ *and the limit map* $G \to G/N$ *is the quotient morphism.*

Example of a projective system that is not strict:
Exercise. Let G be a compact group and $\mathcal{F}$ a filter basis of closed subgroups. Then

$$\{H, H \in \mathcal{F}; f_{HK}\colon K \to H,\ H, K \in \mathcal{N}, K \subseteq H,\}$$

$f_{HK}(k) = k$, is a projejctive system. Show that $\lim_{H\in\mathcal{F}} H = \bigcap_{H\in\mathcal{F}} H = \bigcap \mathcal{F}$

Program for today.

Proof of Proposition B. Character groups of abelian groups as projective limits.