May 8, 2006
Introduction to Compact Groups

We verified that the evaluation morphism is a natural morphism having the following universal property:
For each morphism $f\colon A \to \widehat{G}$ where A is an abelian group and G is a compact group there is a unique morphism $f'\colon G \to \widehat{A}$ such that $f = \widehat{f'} \circ \eta_A$.
There is a natural isomorphism of abelian groups

$$f \mapsto f' : \mathrm{Hom}(A, \widehat{G}) \to \mathrm{Hom}(G, \widehat{A}).$$

This is independent of information whether η_A is an isomorphism or not.
A similar piece of information arises by exchanging abelian groups and compact abelian groups.
There is an immediate corollary:
For each abelian group A the composition

$$\widehat{A} \xrightarrow{\eta_{\widehat{A}}} \widehat{\widehat{\widehat{A}}} \xrightarrow{\widehat{\eta_A}} \widehat{A}$$

is the identity morphism of $\widehat{A}$.

Recall: If $f\colon A \to B$ and $g\colon B \to A$ satisfy $g \circ f = \mathrm{id}_A$, then $B = \ker g \oplus \mathrm{im} f$: A is a homomorphic retract of B.

We defined the concept of a projective system

$$\{G_j, j \in J; f_{jk}\colon G_k \to G_j \text{ for } j \leq k\}$$

and its limit $L = \lim_{j\in J} G_j$, namely, the set of all $(g_j)_{j\in J} \in \prod G_j$ such that $f_{jk}(g_k) = g_j$ for all $j \leq k$. Recall that $f_{ij} \circ f_{jk} = f_{ik}$ and $f_{jj} = \mathrm{id}$.

Program for today.
Projective Limits. Character groups of abelian groups as projective limits.