April 27, 2006
Introduction to Compact Groups

Reminder: Our website is
"http://www.mathematik.tu-darmstadt.de/lehrmaterial/SS2006/CompGroups/"

If $\{A_j : j \in J\}$ is an arbitrarly family of abelian groups and B is a compact abelian group, then the morphism

$$\Phi\colon \prod_{j\in J} \mathrm{Hom}(A_j, B) \to \mathrm{Hom}(\bigoplus_{j\in J} A_j, B)$$

given by $\Phi((f_j)_{j\in J})((a_j)_{j\in J}) = \sum_{j\in J} f_j(a_j)$ is an isomorphism of compact abelian groups.

In particular, taking $B = \mathbb{T}$ we see that the character group of an arbitrary direct sum of abelian groups is the product of the character groups.

For instance $(\mathbb{Z}^J)\widehat{\ } = \mathbb{T}^J$.

Program for today.

Prove that for any abelian group A and any nonzero $a \in A$ there is a character $\chi \in \widehat{A}$ such that $\chi(a) \neq 0$.