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Introduction to Compact Groups

Our website is
"http://www.mathematik.tu-darmstadt.de/
lehrmaterial/SS2006/CompGroups/"

If A and B are abelian groups, then $\mathrm{Hom}(A,B) \subseteq B^A$, the subset of all homomorphisms, is a subgroup of B^A. If B is a topological Hausdorff group, then $\mathrm{Hom}(A,B)$ is closed. If B is compact, then $\mathrm{Hom}(A,B)$ is a compact abelian group.
If A is an abelian group, then the compact abelian group $\hat{A} = \mathrm{Hom}(A,\mathbb{T})$ is called the character group of A, and its elements $\chi\colon A \to \mathbb{T}$ are called characters.
Recall $\mathbb{Z}(n) = \mathbb{Z}/n\mathbb{Z}$. We observed $\widehat{\mathbb{Z}} \cong \mathbb{T}$ and $\mathbb{Z}(n)\widehat{\ } \cong \mathbb{Z}(n)$.

Program for today.
Character groups of direct sums of abelian groups.
Does any abelian group have characters?
If so—how many?