Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

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9. Tutorial Analysis II for MCS
Summer Term 2006

(T9.1)

Let $X \subseteq Y$ be a subspace of a complete metric space Y. Show that the following are equivalent.

(i) X is closed in Y.

(ii) X is complete.

(This is Proposition 6.15 in the handouts.)

(T9.2)

Let V and W be normed spaces over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Recall that we denote by $L(V, W)$ the set of continuous linear transformations from V to W. Prove the following.

(i) $L(V, W)$ is a vector space over $\mathbb{K}$ with respect to pointwise vector addition and scalar multiplication. That is, $L(V, W)$ is a vector space over $\mathbb{K}$ if we define addition

$$(S, T) \mapsto S + T : L(V, W) \times L(V, W) \to L(V, W)$$

on $L(V, W)$ by

$$(S + T)(x) := S(x) + T(x),$$

for all $S, T \in L(V, W)$ and $x \in V$, and scalar multiplication

$$(r, T) \mapsto r \cdot T : \mathbb{K} \times L(V, W) \to L(V, W)$$

on $L(V, W)$ by

$$(r \cdot T)(x) := r \cdot T(x),$$

for all $r \in \mathbb{K}$, $T \in L(V, W)$ and $x \in V$.

(ii) The function $\| \cdot \| : L(V, W) \to \mathbb{R}$, defined for every $T \in L(V, W)$ by

$$\|T\| := \sup\{\|T(x)\| : x \in V, \|x\| \leq 1\},$$

is a norm on $L(V, W)$. Moreover, $\|T(x)\| \leq \|T\| \cdot \|x\|$ for all $x \in V$.

(iii) If W is a Banach space, then $L(V, W)$ is also a Banach space.

(This is Theorem 6.26 in the handouts.)

Orientation Colloquium

The Department of Mathematics' Research Groups present themselves.

Monday, 19.06.2006 – 16:15-17:15 – S207/109

Dr. rer. nat. Patrizio Neff

FG Analysis

"Exkursionen in die nichtlineare Elastizität und Plastizität – Herausforderungen an die angewandte Mathematik"

After the talk there will be a relaxed get-together (coffee, tea and biscuits) in S215/219, where interested people can discuss the talk and become more acquainted with the lecturer.