Fachbereich Mathematik
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8. Tutorial Analysis II for MCS
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(T8.1)

(i) Let $a < b \in \mathbb{R}$ and let $f, g :]a, b] \to [0, \infty[$ be functions which for each $0 < \varepsilon < b - a$ are Riemann integrable on $[a + \varepsilon, b]$. Assume that there exists $a < a_0 \leq b$ such that $f(x) \leq g(x)$ for $x \in]a, a_0]$. Show that if the improper integral $\int_a^b g$ exists then $\int_a^b f$ exists as well.

(ii) Let $a \in \mathbb{R}$ and let $f, g : [a, \infty[\to [0, \infty[$ be functions which for each $b > a$ are Riemann integrable on $[a, b]$. Assume that there exists $a_0 \geq a$ such that $f(x) \leq g(x)$ for $x \geq a_0$. Show that if the improper integral $\int_a^\infty g$ exists then $\int_a^\infty f$ exists as well.

(T8.2) (The Gamma Function)

(i) Prove that the improper integral

$$\int_0^\infty t^{x-1} e^{-t} \, \mathrm{d}t$$

exists for $x > 0$.

(ii) For $x > 0$ we define

$$\Gamma(x) := \int_0^\infty t^{x-1} e^{-t} \, \mathrm{d}t.$$

Prove that $\Gamma(n + 1) = n!$ for all $n \in \mathbb{N}$ and that $x\Gamma(x) = \Gamma(x + 1)$ for all $x \in]0, \infty[$.

Hint: Apply integration by parts.