

7. Tutorial Analysis II for MCS Summer Term 2006

(T7.1)

Let $f, g : [a, b] \rightarrow \mathbb{R}$ be such that $\{x : x \in [a, b] \wedge f(x) \neq g(x)\}$ is finite. Show that f is integrable if and only if g is integrable, and that we in this case have $\int f = \int g$.

(T7.2)

Let $f : [a, \infty[\rightarrow [0, \infty[$ be an increasing function. Prove the following.

(i)

$$\lim_{x \rightarrow \infty} f(x) = \begin{cases} \|f\| & \text{if } f \text{ is bounded,} \\ \infty & \text{otherwise.} \end{cases}$$

(ii) f is bounded if and only if $(f(n))_{n \geq a}$ is bounded.