Fachbereich Mathematik
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TECHNISCHE
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DARMSTADT

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6. Tutorial Analysis II for MCS
Summer Term 2006

(T6.1)

Let $a < b \in \mathbb{R}$ and $C([a,b])$ be the set of all continuous functions $f : [a,b] \to \mathbb{R}$. For any $p \in \mathbb{R}, p \geq 1$, and for any $f \in C([a,b])$, define

$$\|f\|_p := \left(\int_a^b |f|^p \, dx \right)^{\frac{1}{p}}.$$

Prove that $\|\cdot\|_p$ is a norm on the set $C([a,b])$, that is, that for any $f, g \in C([a,b])$, and for any $\lambda \in \mathbb{R}$ the following hold:

(i) $\|f\|_p \geq 0$ and ($\|f\|_p = 0$ if and only if $f = 0$).

(ii) $\|\lambda f\|_p = |\lambda| \cdot \|f\|_p$.

(iii) $\|f + g\|_p \leq \|f\|_p + \|g\|_p$.

Hint for (iii): Use the following intermediate steps:

(a) Let $a, b \geq 0$, and $p, q \in \mathbb{R}, p, q > 1$ be such that $\frac{1}{p} + \frac{1}{q} = 1$. Prove the following inequality:

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}. \tag{1}$$

(b) Prove the **Hölder Inequality**: Let $p, q > 1$ be such that $\frac{1}{p} + \frac{1}{q} = 1$. Then for any functions $f, g \in C([a,b])$,

$$\int_a^b |fg| \, dx \leq \|f\|_p \cdot \|g\|_q. \tag{2}$$

(c) Prove the **Minkowski Inequality**: For any $p \geq 1$ and for any functions $f, g \in C([a,b])$,

$$\|f + g\|_p \leq \|f\|_p + \|g\|_p. \tag{3}$$

Orientation Colloquium

The Department of Mathematics' Research Groups present themselves.

Monday, 29.05.2006 – 16:15-17:15 – S207/109

Prof. Dr. Burkhard Kümmerer

FG Algebra, Geometrie und Funktionalanalysis

"Im Dreiländereck Funktionalanalysis – Stochastik – Mathematische Physik"

After the talk there will be a relaxed get-together (coffee, tea and biscuits) in S215/219, where interested people can discuss the talk and become more acquainted with the lecturer.