Fachbereich Mathematik
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3. Tutorial Analysis II for MCS
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(**T3.1**) Calculate the derivative of the tangent function

$$\tan : \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[\to \mathbb{R}, \qquad \tan(x) := \frac{\sin(x)}{\cos(x)}.$$

Show that this function is strictly isotone and surjective.

(**T3.2**) Argue that

$$
\begin{aligned}
\arcsin : \left] -1, 1 \right[&\to \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[, \\
\arccos : \left] -1, 1 \right[&\to \left] 0, \pi \right[, \\
\arctan : \mathbb{R} &\to \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[,
\end{aligned}
$$

the inverse functions of respectively

$$
\begin{aligned}
\sin : \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[&\to \left] -1, 1 \right[, \\
\cos : \left] 0, \pi \right[&\to \left] -1, 1 \right[, \\
\tan : \left] -\frac{\pi}{2}, \frac{\pi}{2} \right[&\to \mathbb{R},
\end{aligned}
$$

are well defined. Prove that the former are differentiable, and that their derivatives satisfy

$$
\begin{aligned}
\arcsin'(x) &= \frac{1}{\sqrt{1 - x^2}}, \\
\arccos'(x) &= \frac{-1}{\sqrt{1 - x^2}}, \\
\arctan'(x) &= \frac{1}{1 + x^2}.
\end{aligned}
$$

(**T3.3**) Find an antiderivative of the function $x \mapsto \frac{1}{\sqrt{1-x^2}} :] -1, 1[\to \mathbb{R}$, and an antiderivative of the function $x \mapsto \frac{1}{1+x^2} : \mathbb{R} \to \mathbb{R}$.