Fachbereich Mathematik
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2. Tutorial Analysis II for MCS
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(T2.1)

Let $f : I \to \mathbb{R}$ be a function of class C^n on an interval $I \subseteq \mathbb{R}$ and let a be an inner point of I. Assume that

$$f'(a) = f''(a) = \ldots = f^{(n-1)}(a) = 0, f^{(n)}(a) \neq 0.$$

Prove the following assertions:

(i) If n is even, then f has a local extremum at a. More precisely, if $f^{(n)}(a) > 0$, then f has a local minimum at a, if $f^{(n)}(a) < 0$, then f has a local maximum at a.

(ii) If n is odd, then f does not have a local extremum at a.

(T2.2)

Prove Corollary 4.12:
Assume that the function $f : U_\rho(0) \to \mathbb{K}$ satisfies the hypotheses of Theorem 4.11. (Here $\mathbb{K}$ stands for either $\mathbb{R}$ or $\mathbb{C}$.) Then all successive derivatives $f^{(k)} : U_\rho(0) \to \mathbb{K}$ exist (recall that $f^{(0)} = f$ and $f^{(k+1)} = (f^{(k)})'$) and

$$f^{(k)}(x) = k! \sum_{n=k}^{\infty} \binom{n}{k} a_n x^{n-k} = k! \sum_{n=0}^{\infty} \binom{n+k}{k} a_{n+k} x^n.$$

(T2.3) Supplementary exercise.

Prove that if $I \subseteq \mathbb{R}$ is an interval and $f : I \to \mathbb{R}$ is differentiable, then the image $f'(I)$ of the derivative is an interval.