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1. Tutorial Analysis II for MCS Summer Term 2006

(T1.1) Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function. Prove that if the derivative of f has at most k distinct zeros, then f has at most $k + 1$ distinct zeros.

Hint: Use Rolle's Theorem.

(T1.2) Let $A \subseteq \mathbb{R}$ be an open set and let $f : A \rightarrow \mathbb{R}$ be a continuous function, which is differentiable on $A \setminus \{x_0\}$ for some $x_0 \in A$. Prove that if $\lim_{\substack{x \rightarrow x_0 \\ x \neq x_0}} f'(x)$ exists, then f is

differentiable at x_0 and $f'(x_0) = \lim_{\substack{x \rightarrow x_0 \\ x \neq x_0}} f'(x)$.