Fachbereich Mathematik
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13. Home work Analysis II for MCS
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(H13.1)

Consider the function $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$f(x, y) = (e^x \cos y, e^x \sin y).$$

(i) Show that f is differentiable and compute its derivative.

(ii) Show that f is locally invertible around every point in $\mathbb{R}^2$.

(iii) Show that f does not have a global inverse.

(H13.2)

Let us consider the following system of equations:

$$\begin{aligned} x^2 + 4y^2 + 9z^2 &= 1 \\ x + y + z &= 0. \end{aligned}$$

(i) Show that this system can be solved uniquely with respect to y and z,

$$\begin{pmatrix} y \\ z \end{pmatrix} = g(x),$$

in a neighborhood of the point $(0, 1/\sqrt{13}, -1/\sqrt{13})$.

(ii) Compute $g'(0)$.

(H13.3)

Let $a \in \mathbb{R}$, and define $h_1, h_2 : \mathbb{R}^2 \to \mathbb{R}$ by

$$h_1(x, y) = x \cos a - y \sin a$$

and

$$h_2(x, y) = x \sin a + y \cos a.$$

We will write $u = h_1(x, y)$ and $v = h_2(x, y)$. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be a function from $\mathbb{R}^2$ into $\mathbb{R}$, and let $g : \mathbb{R}^2 \to \mathbb{R}$ be defined by $g(x, y) = f(h_1(x, y), h_2(x, y))$. Prove that

$$(\frac{\partial g}{\partial x}(x, y))^2 + (\frac{\partial g}{\partial y}(x, y))^2 = (\frac{\partial f}{\partial u}(u, v))^2 + (\frac{\partial f}{\partial v}(u, v))^2.$$