Fachbereich Mathematik
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12. Home work Analysis II for MCS
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(H12.1)

Compute the derivative of the function $f : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$f(x, y, z) = (x \sin(y) \cos(z), x \sin(y) \sin(z), x \cos(y)).$$

(H12.2) Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{x^2+y^2} & (x, y) \neq (0, 0), \\ 0 & (x, y) = (0, 0). \end{cases}$$

Prove the following assertions.

(i) The partial derivatives $\partial_j f(0, 0)$ exist for $j = 1, 2$.

(ii) The directional derivative $D_v f(0, 0)$ does not exist if v is not a multiple of the standard unit vectors e_1, e_2.

(iii) The function f is not differentiable.

(H12.3)

Let $n \in \mathbb{N}$, and let $D \subseteq \mathbb{R}^n$ be open. Let $f : D \to \mathbb{R}$ be differentiable at $a \in \mathbb{R}^n$. Let $v \in \mathbb{R}^n$,

$$v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}.$$

Prove that

$$D_v f(a) = \sum_{i=1}^{n} \frac{\partial f}{\partial x_i}(a) \cdot v_i.$$