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11. Home work Analysis II for MCS
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(H11.1)

Compute the arc length of the following curves.

(i) Let $0 < a < b < \infty$. We define $f : [a, b] \to \mathbb{R}^2$, $f(t) = (t^3, \frac{3}{2}t^2)$.

(ii) We define

$$\gamma : [0, 2\pi] \to \mathbb{R}^2, \qquad t \mapsto ((1 + \cos t)\cos t, (1 + \cos t)\sin t).$$

(H11.2)

We define the following relation for paths. Two paths $f_j : [a_j, b_j] \to X$, $j = 1, 2$, are called *equivalent* if there is a strictly isotone surjective function $\sigma : [a_1, b_1] \to [a_2, b_2]$ such that $f_1 = f_2 \circ \sigma$. That is, if there exists a change of parameters $\sigma : [a_1, b_1] \to [a_2, b_2]$ such that f_1 is obtained from f_2 by σ, and moreover, σ is strictly isotone.

Prove that this relation is indeed an equivalence relation on the set of all curves in a fixed metric space X.

(This is Remark 8.15 in the handouts.)

(H11.3)

Prove the following:

The image of a rectifiable curve in $\mathbb{R}^2$ does not contain the square $[0, 1]^2$.

(Conclude that γ_{sc} as defined in Tutorial 11 is not rectifiable.)

Hint: Let $\delta : [a, b] \to \mathbb{R}^2$ be a rectifiable curve and assume that $Q = [0, 1]^2 \subseteq \delta([a, b])$. Let $n \in \mathbb{N}$ and define a subset $M \subseteq Q$ by

$$M = \left\{ \left(\frac{p}{n}, \frac{q}{n} \right) : 0 \leq p, q \leq n \right\}.$$

Remark that according to our assumption there are points $t_1 < t_2 < \ldots < t_{(n+1)^2}$ of $[a, b]$ such that $\delta(\{t_1, \ldots, t_{(n+1)^2}\}) = M$. Consider a partition P of $[a, b]$ containing the points $t_1 < \ldots < t_{(n+1)^2}$.