Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

21.06.2006

9. Home work Analysis II for MCS
Summer Term 2006

(H9.1)

Let V be a $\mathbb{K}$-vector space, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Suppose that $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent norms on V. Prove that for any subset $A \subseteq V$ the following hold:

(i) $(V, \|\cdot\|_1)$ is a Banach space if and only if $(V, \|\cdot\|_2)$ is a Banach space.

(ii) A is closed in $(V, \|\cdot\|_1)$ if and only if A is closed in $(V, \|\cdot\|_2)$, and A is compact in $(V, \|\cdot\|_1)$ if and only if A is compact in $(V, \|\cdot\|_2)$.

(iii) A is connected in $(V, \|\cdot\|_1)$ if and only if A is connected in $(V, \|\cdot\|_2)$.

(This is a part of Proposition 6.20 in the handouts.)

(H9.2)

If $(x_n)_{n\in\mathbb{N}}$ is a Cauchy sequence in a normed vector space V over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, and if $(\alpha_n)_{n\in\mathbb{N}}$ is a convergent sequence in $\mathbb{K}$, show that the sequence $(\alpha_n x_n)_{n\in\mathbb{N}}$ is Cauchy.

(H9.3)

Let V and W be normed spaces over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Let $\|\cdot\|$ be the operator norm on $L(V, W)$, and let $T \in L(V, W)$. Prove that

$$\|T\| = \sup\{\|T(x)\| : x \in V, \|x\| = 1\}.$$

(This is a part of Proposition 6.28 in the handouts.)

Orientation Colloquium

The Department of Mathematics' Research Groups present themselves.

Monday, 19.06.2006 – 16:15-17:15 – S207/109

Dr. rer. nat. Patrizio Neff

FG Analysis

"Exkursionen in die nichtlineare Elastizität und Plastizität – Herausforderungen an die angewandte Mathematik"

After the talk there will be a relaxed get-together (coffee, tea and biscuits) in S215/219, where interested people can discuss the talk and become more acquainted with the lecturer.