Fachbereich Mathematik
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8. Home work Analysis II for MCS
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(H8.1)

Determine which of the following define norms on $\mathbb{R}^3$.

(i) $\|(x_1, x_2, x_3)\| := x_1^2 + x_2^2 + x_3^2$.

(ii) $\|(x_1, x_2, x_3)\| := |x_1|$.

(iii) $\|(x_1, x_2, x_3)\| := \max\{|x_1|, |x_2|\} + |x_3|$.

(iv) $\|(x_1, x_2, x_3)\| := (\sqrt{|x_1|} + \sqrt{|x_2|} + \sqrt{|x_3|})^2$.

(H8.2)

Let $(V, \|\cdot\|_1)$ and $(W, \|\cdot\|_2)$ be normed vector spaces over $\mathbb{K}$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$.

Recall that $V \times W = \{(v, w) : v \in V \wedge w \in W\}$ becomes a vector space when we define

$$(v_1, w_1) + (v_2, w_2) := (v_1 + v_2, w_1 + w_2),$$

$$r(v_1, w_1) := (rv_1, rw_1),$$

for $v_1, v_2 \in V$, $w_1, w_2 \in W$ and $r \in \mathbb{K}$.

Define $\|\cdot\| : V \times W \to \mathbb{R}$ by $\|(v, w)\| := \|v\|_1 + \|w\|_2$. Show that $\|\cdot\|$ is a norm on $V \times W$.

(H8.3)

Let $(V, \|\cdot\|)$ be a normed space which contains at least two distinct elements. Prove that for $r \in [0, \infty[$ there is an element $x \in V$ with norm r. Conclude that normed spaces which contain at least two distinct elements cannot be bounded.