

5. Home work Analysis II for MCS Summer Term 2006

(H5.1)

Let $f, g : [a, b] \rightarrow \mathbb{R}$ be real functions. Prove the following statements:

- (i) $|f| \geq 0$ and $(|f| = 0 \text{ if and only if } f = 0)$.
- (ii) $|f + g| \leq |f| + |g|$ and $|fg| = |f| \cdot |g|$.
- (iii) $|f| = f^+ + f^-$ and $f = f^+ - f^-$.

(H5.2)

Let $a < b \in \mathbb{R}$ and $f : [a, b] \rightarrow \mathbb{R}$, $f(x) = x^2$. Prove that f is integrable and compute

$$\int_a^b f dx.$$

Hint: Use (T5.1)(ii).

(H5.3)

Let $a < b \in \mathbb{R}$, and let $f, g : [a, b] \rightarrow \mathbb{R}$ be continuous on $[a, b]$ and differentiable on $]a, b[$. Assume that $f(a) \leq g(a)$ and that $f'(x) < g'(x)$ for $x \in]a, b[$. Show that $f(x) < g(x)$ for $x \in]a, b[$.