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4. Home work Analysis II for MCS Summer Term 2006

(H4.1)

Determine the local maxima and minima of the following function:

$$f: \mathbb{R} \rightarrow \mathbb{R}, \quad f(x) := \frac{x}{1+x^2}.$$

(H4.2)

Prove the following version of the rule of Bernoulli and de l'Hôpital:

Let $a \in \mathbb{R}$ and let $f:]a, \infty[\rightarrow \mathbb{R}$ and $g:]a, \infty[\rightarrow \mathbb{R}$ be functions. Assume that:

(1) There is a $M \in \mathbb{R}$ such that f and g are differentiable on $]M, \infty[$, and such that $g(x) \neq 0$ and $g'(x) \neq 0$ for $x > M$.

(2) The limit $l = \lim_{x \rightarrow \infty} \frac{f'(x)}{g'(x)}$ exists.

(3) $\lim_{x \rightarrow \infty} f(x) = 0$ and $\lim_{x \rightarrow \infty} g(x) = 0$.

Then the limit

$$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$$

exists and coincides with l .

(This exercise has with hindsight been modified to make life easier for you. The same holds true for (T4.2).)