

## 2. Home work Analysis II for MCS Summer Term 2006

### (H2.1)

Let us consider the polynomial function  $f: \mathbb{R} \rightarrow \mathbb{R}$ ,  $f(x) = x^n + ax + b$ , where  $a, b \in \mathbb{R}$ ,  $n \in \mathbb{N}$ . Prove the following:

- (i) if  $n$  is even, then  $f$  has at most two zeros,
- (ii) if  $n$  is odd, then  $f$  has at most three zeros.

Hint: Apply (T1.1).

### (H2.2)

Let  $a, b \in ]0, \infty[$  with  $a < b$ , and let  $n \in \mathbb{N}$  with  $n \geq 2$ . Prove that

$$na^{n-1}(b-a) < b^n - a^n < nb^{n-1}(b-a). \quad (1)$$

Hint: Use the Mean Value Theorem.