Fachbereich Mathematik
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14. Exercise sheet Analysis II for MCS
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(G14.1)

(i) Show that the equation $x^3 + y^2 - 2xy = 0$ may be solved uniquely for (x, y) near $(1, 1)$ with respect to x and that the obtained function $x = \varphi(y)$ is continuously differentiable near $y = 1$. Calculate $\varphi'(1)$.

(ii) Show that φ is two times continuously differentiable near $y = 1$ and calculate $\varphi''(1)$.

(iii) Is the equation uniquely solvable with respect to y near $(1, 1)$?

(G14.2)

Prove that the map $F\colon \mathbb{R}^2 \to \mathbb{R}^2$ with

$$F(x, y) = \begin{pmatrix} x^2 - y^2 \\ 2xy \end{pmatrix}$$

is locally invertible for $(x, y) \neq (0, 0)$. Is F also globally invertible? Compute the preimage $F^{-1}(\{(a, b)\})$ of an arbitrary point $(a, b) \in \mathbb{R}^2 \setminus \{(0, 0)\}$.

(G14.3) (Supplementary)

Find the global maximum and minimum of the function

$$f(x, y) = 2x^2 + xy + \frac{5}{4}y^2 - 2x - 2y$$

on the unit square $S = [0, 1] \times [0, 1]$.

Hint: To compute the global extrema of a function f defined on a compact subset K of $\mathbb{R}^n$, you have to compute the local extrema on the interior of K and the global extrema on the boundary of K.

(G14.4) (Supplementary)

Let $f : \mathbb{R}^n \to \mathbb{R}^n$ a continiously differentiable function and $f'(x)$ invertible for all $x \in \mathbb{R}^n$. Prove that f is open, i. e. $f(U)$ is open for each open set $U \subseteq \mathbb{R}^n$.