Fachbereich Mathematik
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13. Exercise sheet Analysis II for MCS

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(G13.1)

Let $f:\mathbb{R}^3 \to \mathbb{R}$, $f(x,y,z) = e^z y + x^2y^2$ and $g:\mathbb{R}\to\mathbb{R}^3$,

$$g(t) = \begin{pmatrix} 2t^2 \\ \sin t \\ e^t \end{pmatrix}.$$

Compute the derivative of $f\circ g$ in two different ways.

(i) Directly by computing $h(t) = f(g(t))$ and differentiating h.

(ii) By using the chain rule.

(G13.2)

Let us consider the function

$$f:\mathbb{R}^2\to\mathbb{R}^2, \quad f(x_1,x_2) := \begin{pmatrix} x_1 + x_2\cos x_1 \\ x_2\, e^{x_1x_2} \end{pmatrix}.$$

Prove that the equation $f(x) = z$ for z near $(0,0)$ possesses a unique solution $x = g(z)$ near $(0,0)$. Show that g is continuously differentiable near $(0,0)$ and compute $g'(0,0)$.

(G13.3)

Let $f:\mathbb{R}^3\to\mathbb{R}^2$,

$$f(x,y,z) = \begin{pmatrix} z^2 + xy - 2 \\ z^2 + x^2 - y^2 - 1 \end{pmatrix}.$$

Remark that

$$f(1,1,1) = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Consider the equations

$$\begin{aligned} z^2 + xy - 2 &= 0 \\ z^2 + x^2 - y^2 - 1 &= 0, \end{aligned}$$

and find for some neighborhood U of $z = 1$ in $\mathbb{R}$ a curve of solutions $\gamma : U \to \mathbb{R}^2$, $\gamma(z) = \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$, passing through $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$ at $z = 1$. Prove that γ is continuously differentiable and compute γ'.