Fachbereich Mathematik
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11. Exercise sheet Analysis II for MCS
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(G11.1)

Compute the arc length of the following paths.

(i) $f : [0, 2\pi] \to \mathbb{R}^3$, $f(t) = (r \cos t, r \sin t, ct)$, where $r, c > 0$.

(ii) $g : [0, 2\pi] \to \mathbb{R}^2$, $g(t) = (t - \sin t, 1 - \cos t)$.

(G11.2)

Let (X, d) be a metric space, and let $a \leq b \in \mathbb{R}$ and $c \leq d \in \mathbb{R}$. Let $\gamma' : [c, d] \to X$ be a path obtained from a path $\gamma : [a, b] \to X$ by a change of parameter. Prove that $L(\gamma) = L(\gamma')$.

(This is Proposition 8.13 in the handouts.)

(G11.3)

Let (X, d) be a metric space, and let $a \leq b \in \mathbb{R}$. Let $\gamma : [a, b] \to X$ be a path in X. Prove that for all $c \in [a, b]$ we have

$$L(\gamma) = L(\gamma|_{[a,c]}) + L(\gamma|_{[c,b]}).$$

(Recall that $\gamma|_{[a,c]} : [a, c] \to X$ is the restriction of γ to $[a, c]$, i.e. $\gamma|_{[a,c]}(x) = \gamma(x)$ for $x \in [a, c]$.)

(This is Proposition 8.17 in the handouts.)