Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

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9. Exercise sheet Analysis II for MCS
Summer Term 2006

(G9.1)

Let V be a $\mathbb{K}$-vector space, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Recall that we say that a norm $\|\cdot\|_1$ on V is *equivalent* to a norm $\|\cdot\|_2$ on V if there exist positive numbers $c, C \in]0, \infty[$ such that

$$(\forall x \in V)(c\|x\|_1 \leq \|x\|_2 \leq C\|x\|_1).$$

Prove that this relation is an equivalence relation on the set of all norms on V.

(This is Remark 6.19 in the handouts.)

(G9.2)

Let V be a $\mathbb{K}$-vector space, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$. Suppose that $\|\cdot\|_1$ and $\|\cdot\|_2$ are equivalent norms on V. Prove that for any subset $A \subseteq V$, $x \in V$ and any sequence $(x_n)_{n\in\mathbb{N}}$ in V the following hold:

(i) $(x_n)_{n\in\mathbb{N}}$ is Cauchy in $(V, \|\cdot\|_1)$ if and only if $(x_n)_{n\in\mathbb{N}}$ is Cauchy in $(V, \|\cdot\|_2)$.

(ii) $\lim_{n\to\infty} x_n = x$ in $(V, \|\cdot\|_1)$ if and only if $\lim_{n\to\infty} x_n = x$ in $(V, \|\cdot\|_2)$.

(iii) A is open in $(V, \|\cdot\|_1)$ if and only if A is open in $(V, \|\cdot\|_2)$, and A is bounded in $(V, \|\cdot\|_1)$ if and only if A is bounded in $(V, \|\cdot\|_2)$.

(This is a part of Proposition 6.20 in the handouts.)

(G9.3)

Let $a < b \in \mathbb{R}$ and let $C([a,b])$ be the $\mathbb{R}$-vector space of all continuous functions $f : [a,b] \to \mathbb{R}$. Recall that we for any $1 \leq p < \infty$ can define a norm $\|\cdot\|_p : C([a,b]) \to \mathbb{R}$ on $C([a,b])$ by letting

$$\|f\|_p := \left(\int_a^b |f|^p\right)^{1/p}.$$

Let $I([a,b])$ be the $\mathbb{R}$-vector space of all Riemann integrable functions $f:[a,b]\to\mathbb{R}$. Define $\|\cdot\|_p : I([a,b])\to\mathbb{R}$ for any $1\le p<\infty$ by

$$\|f\|_p := \left(\int_a^b |f|^p\right)^{1/p}.$$

Show that $\|\cdot\|_p$ is not a norm on $I([a,b])$.

Orientation Colloquium

The Department of Mathematics' Research Groups present themselves.

Monday, 19.06.2006 – 16:15-17:15 – S207/109

Dr. rer. nat. Patrizio Neff

FG Analysis

"Exkursionen in die nichtlineare Elastizität und Plastizität – Herausforderungen an die angewandte Mathematik"

After the talk there will be a relaxed get-together (coffee, tea and biscuits) in S215/219, where interested people can discuss the talk and become more acquainted with the lecturer.