Fachbereich Mathematik
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8. Exercise sheet Analysis II for MCS
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(G8.1)

For $x = (x_1, \ldots, x_n) \in \mathbb{K}^n$, $\mathbb{K} \in \{\mathbb{R}, \mathbb{C}\}$, define

$$\|x\|_\infty := \max\{|x_1|, \ldots, |x_n|\}$$

and

$$\|x\|_p := \left(\sum_{i=1}^{n} |x_i|^p \right)^{1/p},$$

for $1 \leq p < \infty$.

(i) Prove that $\|\cdot\|_\infty$ and $\|\cdot\|_p$, for $1 \leq p < \infty$, are norms on $\mathbb{K}^n$.

(ii) Draw the closed unit balls in $(\mathbb{R}^2, \|\cdot\|_\infty)$, $(\mathbb{R}^2, \|\cdot\|_1)$ and $(\mathbb{R}^2, \|\cdot\|_2)$.
Recall that for a normed space $(V, \|\cdot\|)$ the closed unit ball is

$$B_1(0) = \{x \in V : \|x\| \leq 1\}.$$

(G8.2)

Which of the following define norms on $\mathbb{R}^3$?

(i) $\|(x_1, x_2, x_3)\| = x_1 + x_2 + x_3$.

(ii) $\|(x_1, x_2, x_3)\| = |x_1| + |x_2| + 2|x_3|$.

(iii) $\|(x_1, x_2, x_3)\| = 2(x_1^2 + x_2^2 + x_3^2)^{1/2}$.

(G8.3)

(i) Show that if $\|\cdot\|_1$ and $\|\cdot\|_2$ are two norms on a vector space V, then $\|\cdot\| : V \to \mathbb{R}$ defined by

$$\|v\| := \|v\|_1 + \|v\|_2$$

for $v \in V$ is also a norm.

(ii) If $\|\cdot\| : V \to \mathbb{R}$ is a norm on the vector space V, for which $a \in \mathbb{R}$ is $a\|\cdot\|$ a norm?