

7. Exercise sheet Analysis II for MCS Summer Term 2006

(G7.1)

Compute the following integrals.

(i)

$$\int_0^{1/2} \arcsin x \, dx.$$

(ii)

$$\int_0^1 \frac{6x^2 + 4}{x^3 + 2x + 1} \, dx.$$

(iii)

$$\int_a^b \frac{1}{1 - x^2} \, dx,$$

where $a < b \in \mathbb{R}$ and $-1, 1 \notin [a, b]$.

(G7.2)

Let $a < b \in \mathbb{R}$ and let $I_m := \int_a^b \sin^m x \, dx$ for $m \in \mathbb{N}_0$. Give a formula which expresses I_m for $m > 1$ in terms of I_n for $n < m$.

(G7.3)

Let $a \in]0, \infty[$ and let F be an antiderivative of the continuous function $f : [-a, a] \rightarrow \mathbb{R}$ with $F(0) = 0$. Prove

- (i) If f is odd, i.e. $f(x) = -f(-x)$ for $x \in [-a, a]$, then F is even, i.e. $F(x) = F(-x)$ for $x \in [-a, a]$.
- (ii) If f is even, then F is odd.