Fachbereich Mathematik
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TECHNISCHE UNIVERSITÄT DARMSTADT

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6. Exercise sheet Analysis II for MCS

Summer Term 2006

(G6.1)

Prove that the characteristic function of the rational numbers in the unit interval, i.e. the function $f : [0,1] \to \mathbb{R}$ defined by

$$f(x) = \begin{cases} 1 & \text{if } x \in \mathbb{Q}, \\ 0 & \text{otherwise,} \end{cases}$$

is not Riemann integrable.

(G6.2)

(i) Let $a, b \in \mathbb{R}$ with $a < b$, and let $f : [a,b] \to \mathbb{R}$ be a continuous function such that $\int_a^b f(x)\,dx = 0$. Prove that there is $c \in [a,b]$ such that $f(c) = 0$.

(ii) Let $a < b \in \mathbb{R}$ and let $f : [a,b] \to \mathbb{R}$ be an isotone function. Prove that

$$f(a) \leq \frac{1}{b-a} \int_a^b f(x)\,dx \leq f(b).$$

(G6.3) (Supplementary exercise)

Let $f : [0,1] \to \mathbb{R}$ be a continuous function. Prove that

$$\lim_{n\to\infty} \int_0^1 x^n f(x)\,dx = 0.$$

Orientation Colloquium

The Department of Mathematics' Research Groups present themselves.

Monday, 29.05.2006 – 16:15-17:15 – S207/109

Prof. Dr. Burkhard Kümmerer

FG Algebra, Geometrie und Funktionalanalysis

"Im Dreiländereck Funktionalanalysis – Stochastik – Mathematische Physik"

After the talk there will be a relaxed get-together (coffee, tea and biscuits) in S215/219, where interested people can discuss the talk and become more acquainted with the lecturer.