Fachbereich Mathematik
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18.05.2006

5. Exercise sheet Analysis II for MCS
Summer Term 2006

(G5.1)

Let $f, g : [a, b] \to \mathbb{R}$ be bounded real functions. We define

$$\|f\| = \sup\{|f(x)| : x \in [a, b]\}.$$

Prove the following statements:

(i) $\|f\| \geq 0$ and ($\|f\| = 0$ if and only if $f = 0$).

(ii) $\|f + g\| \leq \|f\| + \|g\|$.

(iii) $\|fg\| \leq \|f\| \cdot \|g\|$. In general $\|fg\| \neq \|f\| \cdot \|g\|$.

(G5.2)

Use the arithmetical-geometrical inequality in (G3.2) (ii) to prove the **Hölder inequality**:

$$\sum_{i=1}^{n} a_i b_i \leq \left(\sum_{i=1}^{n} a_i^p\right)^{1/p} \left(\sum_{i=1}^{n} b_i^q\right)^{1/q},$$

for all $n \in \mathbb{N}$ and for all $a_i, b_i \geq 0$ for $1 \leq i \leq n$, and for all $p, q \in]1, \infty[$ such that $\frac{1}{p} + \frac{1}{q} = 1$.

(G5.3) (Supplementary exercise)

Use the Hölder inequality to prove the **Minkowski inequality**:

$$\left(\sum_{i=1}^{n} (a_i + b_i)^p\right)^{1/p} \leq \left(\sum_{i=1}^{n} a_i^p\right)^{1/p} + \left(\sum_{i=1}^{n} b_i^p\right)^{1/p},$$

for $p \in [1, \infty[$ and for $a_i, b_i \in [0, \infty[$ for $1 \leq i \leq n$.