Fachbereich Mathematik
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4. Exercise sheet Analysis II for MCS
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Minitest

Let $I \subseteq \mathbb{R}$ be an interval, $f\colon I \to \mathbb{R}$ be a differentiable function and let x_0 be an interior point of I. Which of the following statements are correct?
(You should not spend more than 10 minutes on the test.)

- ☐ If f has a local minimum or maximum at x_0, then $f'(x_0) = 0$.
- ☐ If $f'(x_0) = 0$, then f has a local minimum or maximum at x_0.
- ☐ If $f'(x_0) = 0$ and $f''(x_0) \neq 0$, then f has a local minimum or maximum at x_0.
- ☐ If $f'(x_0) = 0$ and $f''(x_0) = 0$, then f does not have a local minimum or maximum at x_0.

(G4.1)

(i) Determine the local maxima and minima of the function

$$f :]0, \infty[\to \mathbb{R}, \quad f(x) = x^x.$$

(ii) Determine any global maxima and minima of the function $f : [0, 2\pi] \to \mathbb{R}$ given by $f(x) = e^x \sin x$.

Recall that for a set X and a function $g : X \to \mathbb{R}$, g attains its global maximum in $a \in X$ if $g(a) = \max g(X)$. In the same way, g attains its global minimum in $a \in X$ if $g(a) = \min g(X)$.

(G4.2) (Leibniz Rule)
Let $D \subseteq \mathbb{R}, n \in \mathbb{N}$, and $f, g\colon D \to \mathbb{R}$ be two n-times differentiable functions. Prove the Leibniz Rule:

$$(fg)^{(n)}(x) = \sum_{k=0}^{n} \binom{n}{k} f^{(k)}(x) g^{(n-k)}(x).$$

Hint: Look at the proof of the binomial formula.