Fachbereich Mathematik
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3. Exercise sheet Analysis II for MCS
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(G3.1) Compute the limit

$$\lim_{\substack{x \to 0 \\ x \neq 0}} \frac{\log(\cos x)}{x^2}.$$

(G3.2)

(i) Prove that for $x > 0$,

$$x^\alpha - \alpha x \leq 1 - \alpha \quad (0 < \alpha < 1). \tag{1}$$

(ii) Prove the arithmetical-geometrical inequality

$$a^\alpha b^\beta \leq \alpha a + \beta b \quad (a, b, \alpha, \beta > 0, \alpha + \beta = 1). \tag{2}$$

(iii) Generalize (2) to

$$\prod_{i=1}^{n} a_i^{\alpha_i} \leq \sum_{i=1}^{n} \alpha_i a_i \quad (n \in \mathbb{N}, a_i, \alpha_i > 0, \sum_{i=1}^{n} \alpha_i = 1). \tag{3}$$

(iv) Prove the arithmetic-geometric inequality

$$\left(\prod_{i=1}^{n} a_i \right)^{\frac{1}{n}} \leq \frac{1}{n} \sum_{i=1}^{n} a_i \quad (a_i > 0, n \in \mathbb{N}). \tag{4}$$