Fachbereich Mathematik
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2. Exercise sheet Analysis II for MCS
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(G2.1)

Let $I \subseteq \mathbb{R}$ be an open interval and let $a \in I$. Let $f : I \to \mathbb{R}$ be smooth, i.e. let f have derivatives $f^{(k)}$ for all $k \in \mathbb{N}$. Prove with the help of Taylor's formula that

$$f(a) = f'(a) = ... = f^{(n)}(a) = 0$$

implies

$$\lim_{x \to a} \frac{f(x)}{(x-a)^n} = 0.$$

(G2.2)

Prove the following inequality with the aid of the Mean Value Theorem:

$$1 + \frac{x}{2\sqrt{1+x}} \leq \sqrt{1+x} \leq 1 + \frac{x}{2}\,, \quad -1 < x < \infty. \tag{1}$$

(G2.3) Supplementary exercise.

Use the previous exercise to conclude that

$$1 + \frac{x}{2+x} \leq \sqrt{1+x} \leq 1 + \frac{x}{2}\,, \quad 0 \leq x < \infty. \tag{2}$$

Use this inequality to estimate $\sqrt{102}$ with accuracy 10^{-3}.