Fachbereich Mathematik
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14. Tutorial Analysis II for MCS
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(T14.1)

(i) Read page 39 of the script (Contraction Mapping Principle).

(ii) Prove the uniqueness part of the theorem.

(iii) Read the proof of the Contraction Mapping Principle (pp. 40–41).

Solution.

(i) None.

(ii) Let $x \neq y$ be two fixed points. Then $d(x, y) = d(T(x), T(y)) \leq K d(x, y)$. This is a contradiction because $K < 1$.

(iii) None.

■

(T14.2)

Let (X, d) be a complete metric space and suppose $T : X \to X$ is a function for which T^N is a contraction for some $N \in \mathbb{N}$. Prove that T has a unique fixed point.

Solution. Applying Banach's Contraction Mapping Principle, T^N has a unique fixed point x. However,

$$T^N(T(x)) = T^{N+1}(x) = T(T^N(x)) = T(x),$$

so $T(x)$ is also a fixed point of T^N. Since the fixed point of T^N is unique, we must have $T(x) = x$, so x is a fixed point of T. Let us prove now that it is unique. If $y \in X$ is such that $T(y) = y$, then $T^N(y) = y$, so (by uniqueness of fixed points of T^N) $y = x$. ■