Fachbereich Mathematik
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3. Tutorial Analysis II for MCS
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(T3.1) Calculate the derivative of the tangent function

$$\tan :]-\frac{\pi}{2}, \frac{\pi}{2}[\to \mathbb{R}, \qquad \tan(x) := \frac{\sin(x)}{\cos(x)}.$$

Show that this function is strictly isotone and surjective.

Solution. See pages 263-264 in Hofmann. ∎

(T3.2) Argue that

$$\begin{aligned} \arcsin :]-1,1[&\to]-\frac{\pi}{2}, \frac{\pi}{2}[, \\ \arccos :]-1,1[&\to]0,\pi[, \\ \arctan : \mathbb{R} &\to]-\frac{\pi}{2}, \frac{\pi}{2}[, \end{aligned}$$

the inverse functions of respectively

$$\begin{aligned} \sin :]-\frac{\pi}{2}, \frac{\pi}{2}[&\to]-1,1[, \\ \cos :]0,\pi[&\to]-1,1[, \\ \tan :]-\frac{\pi}{2}, \frac{\pi}{2}[&\to \mathbb{R}, \end{aligned}$$

are well defined. Prove that the former are differentiable, and that their derivatives satisfy

$$\begin{aligned} \arcsin'(x) &= \frac{1}{\sqrt{1-x^2}}, \\ \arccos'(x) &= \frac{-1}{\sqrt{1-x^2}}, \\ \arctan'(x) &= \frac{1}{1+x^2}. \end{aligned}$$

Solution. See pages 263-264 in Hofmann, but refer to the theorem on page 142 in the handouts instead of Corollary 4.19. Note that in this theorem the requirement that I be closed is superfluous. ∎

(T3.3) Find an antiderivative of the function $x \mapsto \frac{1}{\sqrt{1-x^2}} :]-1,1[\to \mathbb{R}$, and an antiderivative of the function $x \mapsto \frac{1}{1+x^2} : \mathbb{R} \to \mathbb{R}$.

Solution. See pages 263-264 in Hofmann. ∎