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13. Homework Analysis II for MCS
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(H13.1) Solution.

(i) We have

$$J_f(x,y) = \begin{pmatrix} e^x \cos y & -e^x \sin y \\ e^x \sin y & e^x \cos y \end{pmatrix}.$$

Since all the partial derivatives are continuous it follows that f is differentiable, and that $J_f(x,y)$ is the associated matrix of $f'(x,y)$.

(ii) We have

$$\det J_f(x,y) = e^{2x}\cos^2 y + e^{2x}\sin^2 y = e^{2x} \neq 0$$

for all $(x,y) \in \mathbb{R}^2$. Thus f is locally invertible around every point of $\mathbb{R}^2$, by the Inverse Function Theorem.

(iii) Since $f(x,y) = f(x, y+2\pi)$ for $(x,y) \in \mathbb{R}^2$, we conclude that f is not injective. Thus f does not have a global inverse.

$\square$

(H 13.3) Solution.

By the chain rule we have

$$\frac{\partial g}{\partial x}(x,y) = \frac{\partial f}{\partial u}(u,v) \cdot \frac{\partial h_1}{\partial x}(x,y) + \frac{\partial f}{\partial v}(u,v) \cdot \frac{\partial h_2}{\partial x}(x,y)$$

$$= \frac{\partial f}{\partial u}(u,v) \cdot \cos a + \frac{\partial f}{\partial v}(u,v) \cdot \sin a,$$

$$\frac{\partial g}{\partial y}(x,y) = \frac{\partial f}{\partial u}(u,v) \cdot \frac{\partial h_1}{\partial y}(x,y) + \frac{\partial f}{\partial v}(u,v) \cdot \frac{\partial h_2}{\partial y}(x,y)$$

$$= \frac{\partial f}{\partial u}(u,v) \cdot (-\sin a) + \frac{\partial f}{\partial v}(u,v) \cdot \cos a.$$

Thus

$$\left(\frac{\partial g}{\partial x}(x,y)\right)^2 + \left(\frac{\partial g}{\partial y}(x,y)\right)^2 =$$

$$\left(\frac{\partial f}{\partial u}(u,v)\right)^2 \cos^2 a + \left(\frac{\partial f}{\partial v}(u,v)\right)^2 \sin^2 a$$

$$+ \left(\frac{\partial f}{\partial u}(u,v)\right)^2 \sin^2 a + \left(\frac{\partial f}{\partial v}(u,v)\right)^2 \cos^2 a$$

$$= \left(\frac{\partial f}{\partial u}(u,v)\right)^2 + \left(\frac{\partial f}{\partial v}(u,v)\right)^2.$$

$\square$